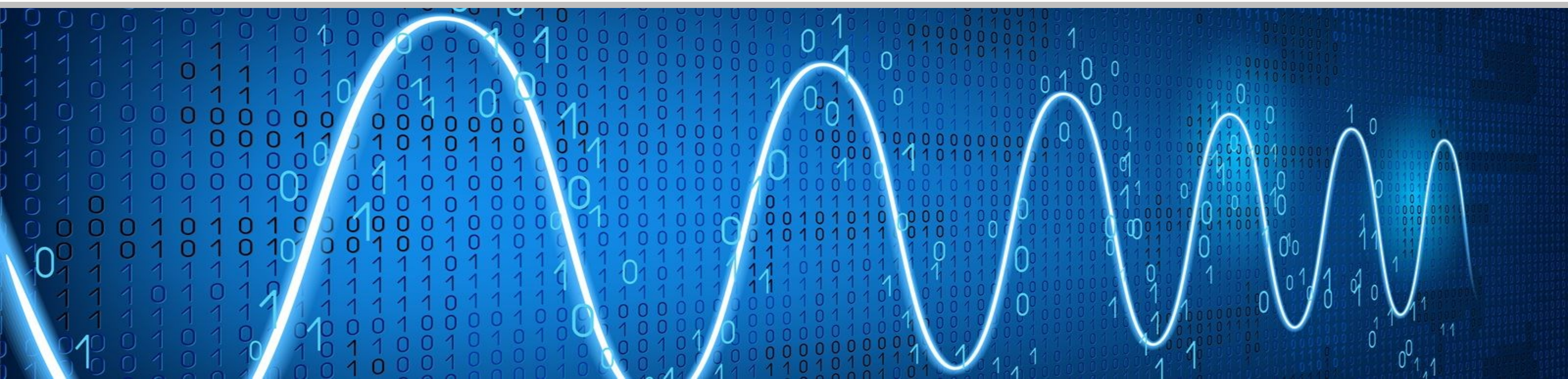
Digital Signal Processing

Lab 03: Signal Representation and Modeling

Abdallah El Ghamry



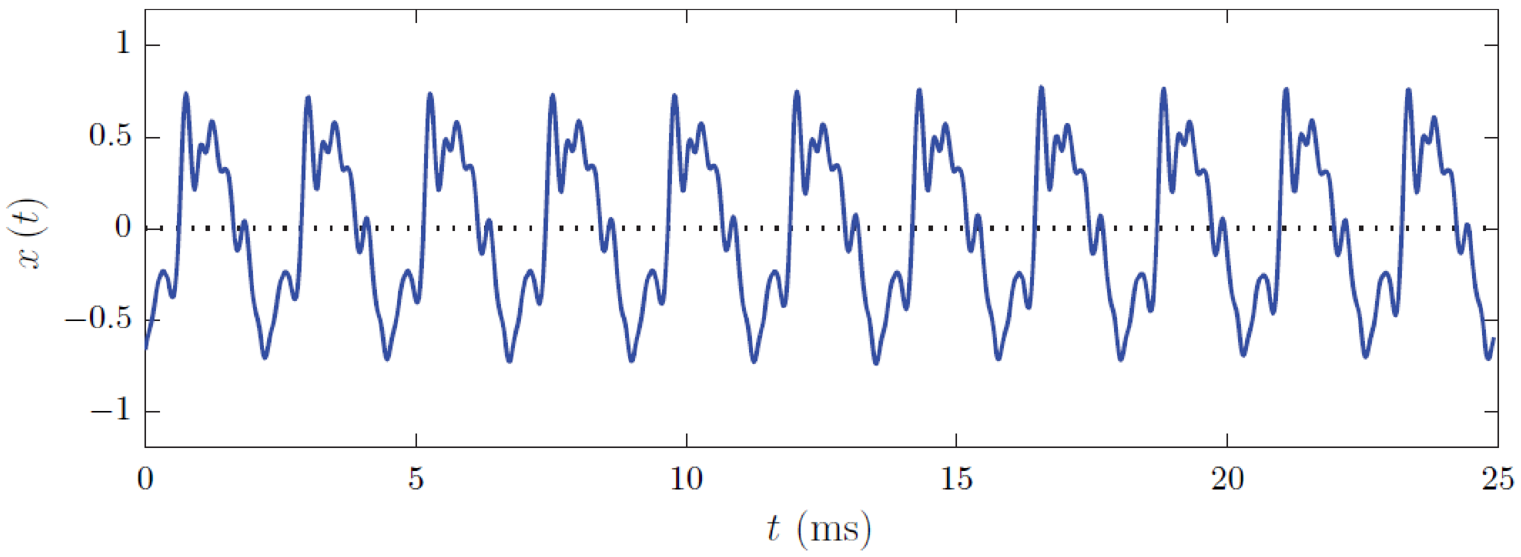
Signal Representation and Modeling

The purpose of this lab is to

- Understand the concept of a signal and how to work with mathematical models of signals.
- Discuss fundamental signal types and signal operations used in the study of signals and systems.
- Experiment with methods of simulating continuous- and discrete-time signals with MATLAB.

Mathematical Modeling of Signals

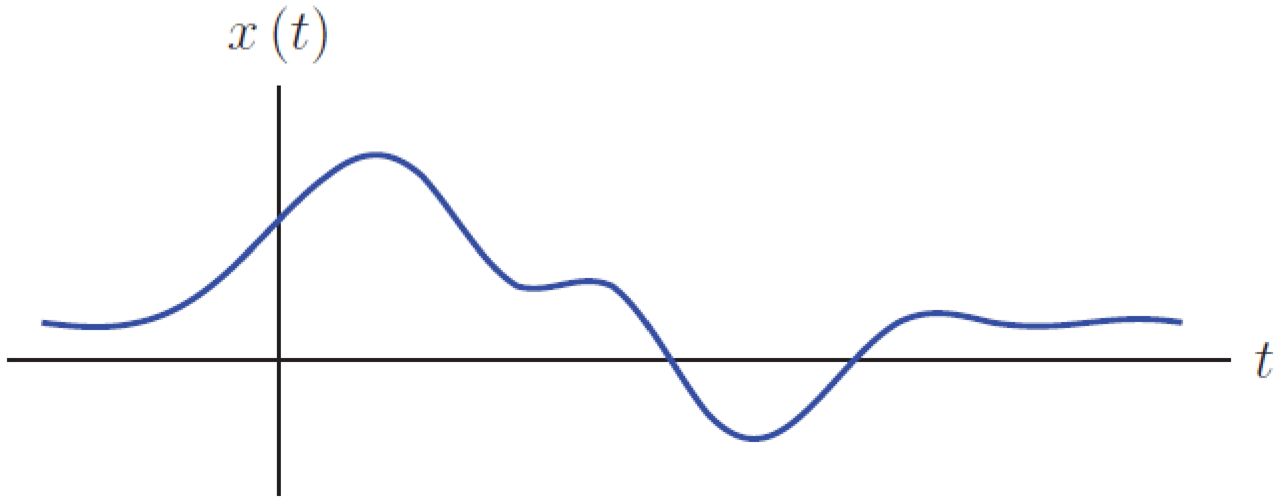
- The mathematical model for a signal is in the form of a formula, function, algorithm or a graph that approximately describes the time variations of the physical signal.



A segment from the sound of a violin.

Continuous-Time Signals

- Consider $x(t)$, a mathematical function of time chosen to approximate the strength of the physical quantity at the time instant t.
- In this relationship t is the independent variable, and x is the dependent variable.
- The signal $x(t)$ is referred to as a continuous-time signal or an analog signal.



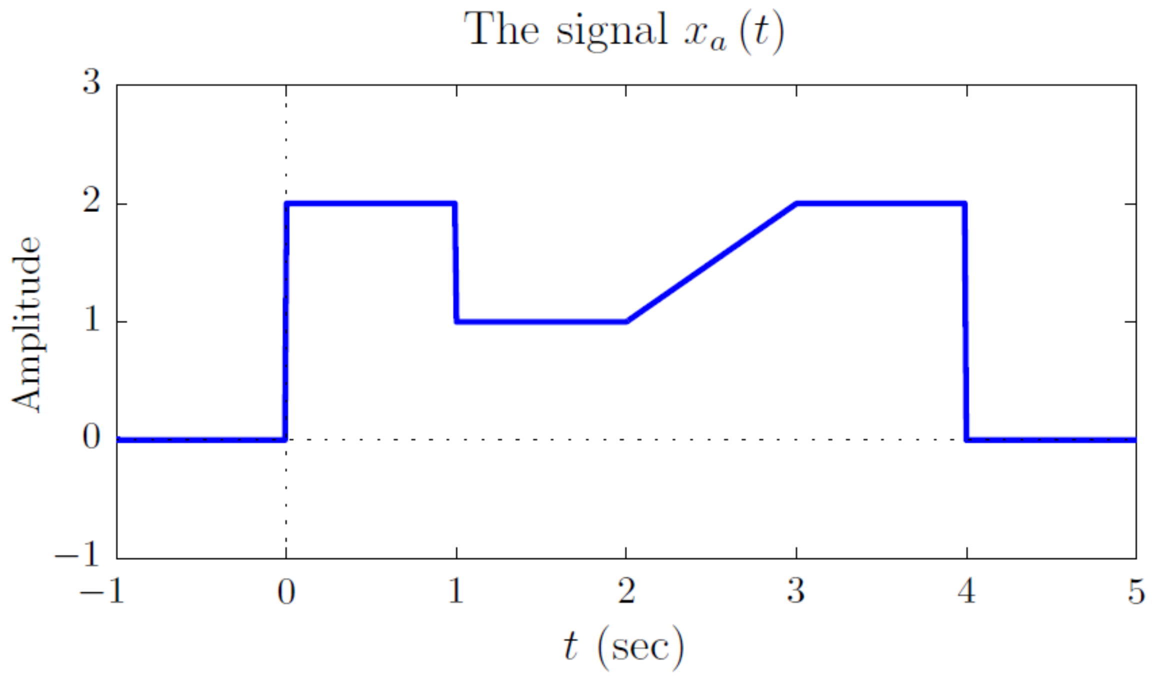
Continuous-Time Signals: Problem 1.1 (a)

1.1. Sketch and label each of the signals defined below:

a.
$$x_a(t) = \begin{cases} 0\,, & t < 0 \ \text{ or } \ t > 4 \\ 2\,, & 0 < t < 1 \\ 1\,, & 1 < t < 2 \\ t - 1\,, & 2 < t < 3 \\ 2\,, & 3 < t < 4 \end{cases}$$

Continuous-Time Signals: Problem 1.1 (a) – Solution

$$x_a(t) = \begin{cases} 0\,, & t < 0 \;\text{ or }\; t > 4 \\ 2\,, & 0 < t < 1 \\ 1\,, & 1 < t < 2 \\ t-1\,, & 2 < t < 3 \\ 2\,, & 3 < t < 4 \end{cases}$$



Continuous-Time Signals: Problem 1.2

1.2. Consider the signals shown in Fig. P.1.2. For each signal write the analytical description in segmented form similar to the descriptions of the signals in Problem 1.1.

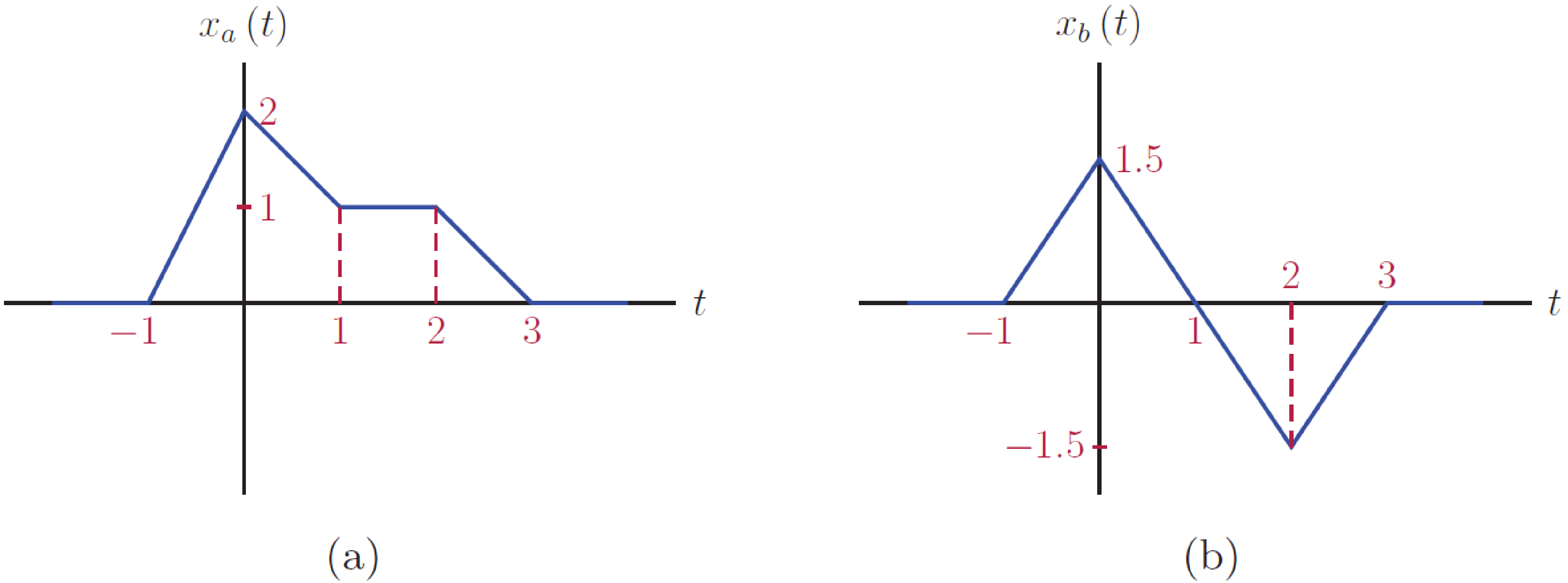
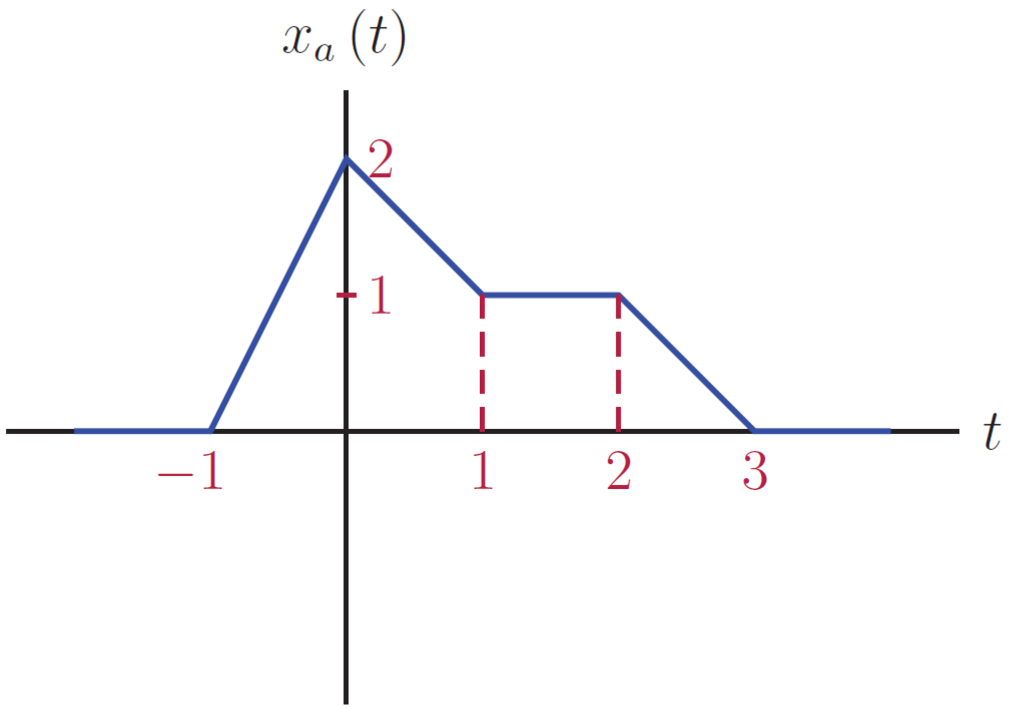


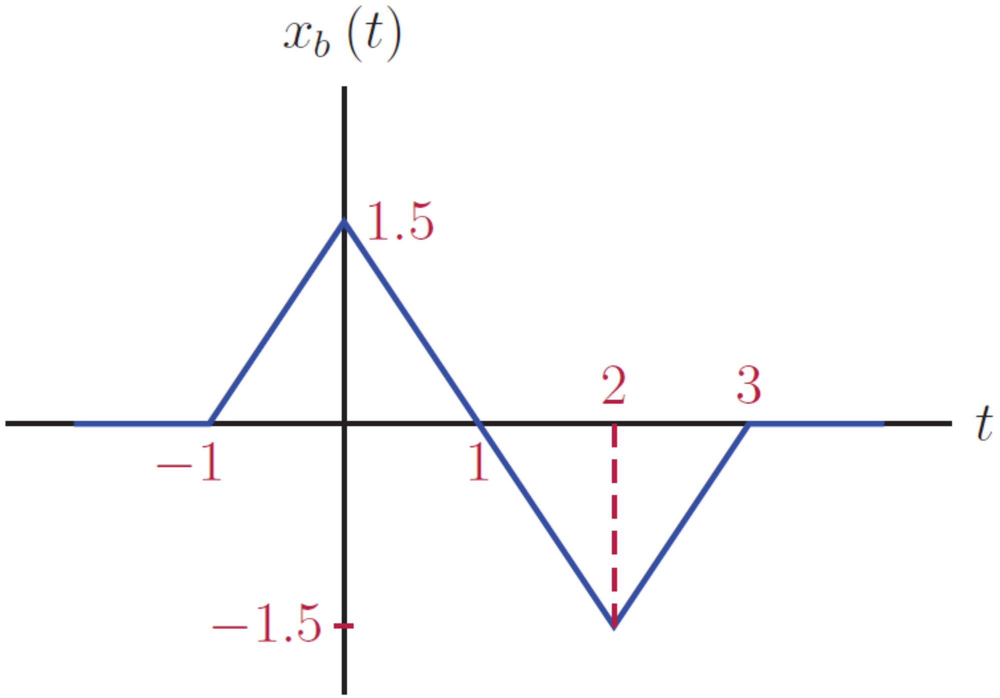
Figure P. 1.2

Continuous-Time Signals: Problem 1.2 (a) – Solution



$$x_a(t) = \begin{cases} 0\ , & t < -1 \text{ or } t > 3 \\ 2t+2\ , & -1 < t < 0 \\ -t+2\ , & 0 < t < 1 \\ 1\ , & 1 < t < 2 \\ -t+3\ , & 2 < t < 3 \end{cases}$$

Continuous-Time Signals: Problem 1.2 (b) – Solution

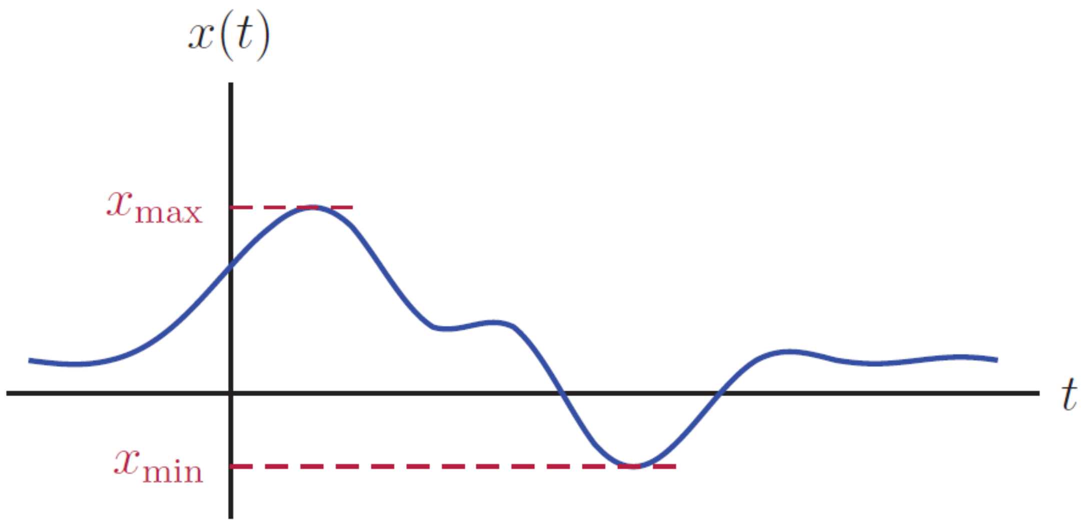


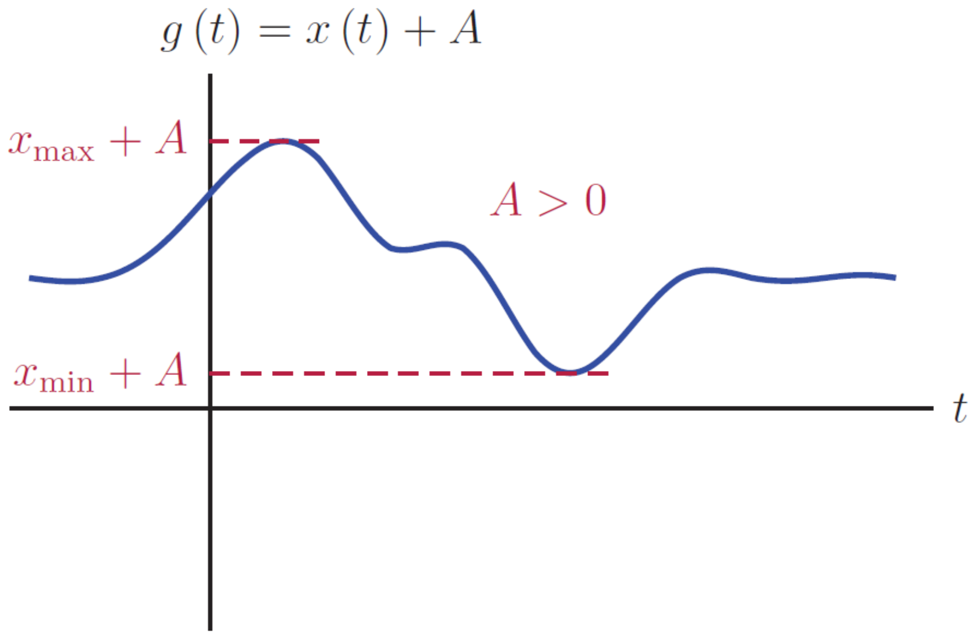
$$x_b(t) = \begin{cases} 0\ , & t < -1 \text{ or } t > 3 \\ 1.5t + 1.5\ , & -1 < t < 0 \\ -1.5t + 1.5\ , & 0 < t < 2 \\ 1.5t - 4.5\ , & 2 < t < 3 \end{cases}$$

Signal Operations: Addition of a Constant Offset

- Addition of a constant offset A to the signal $x(t)$ is expressed as

$$g(t) = x(t) + A$$





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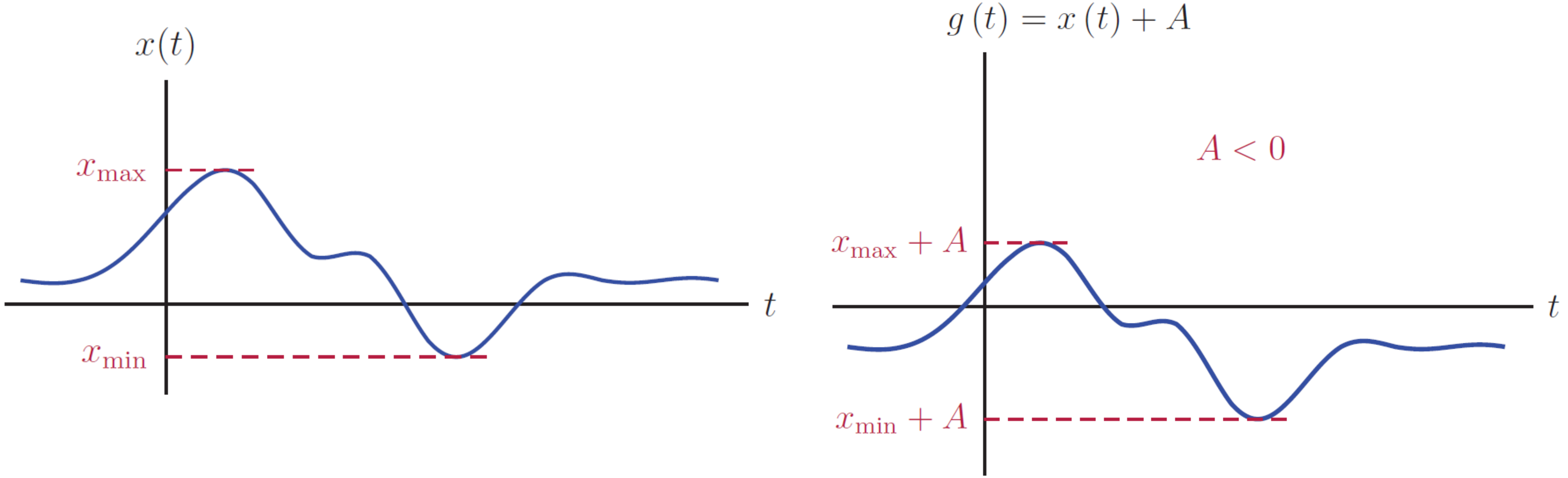
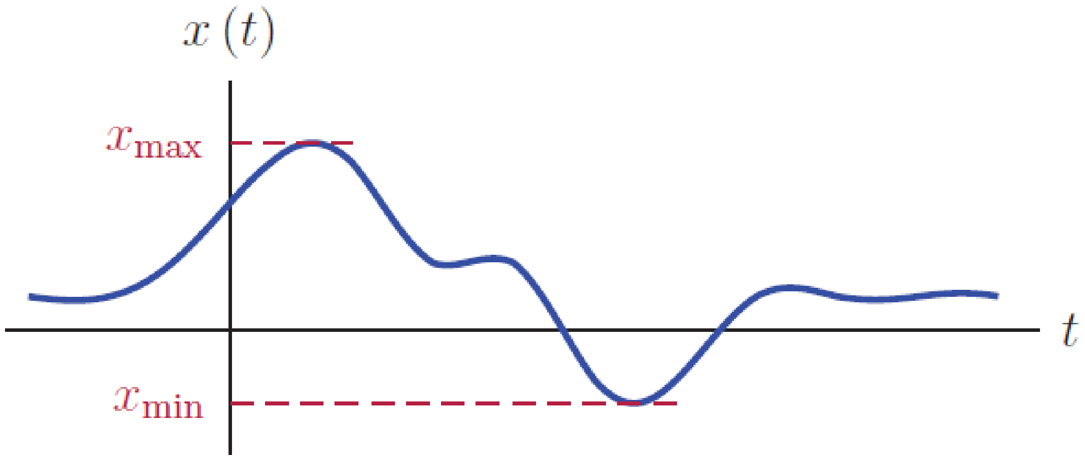
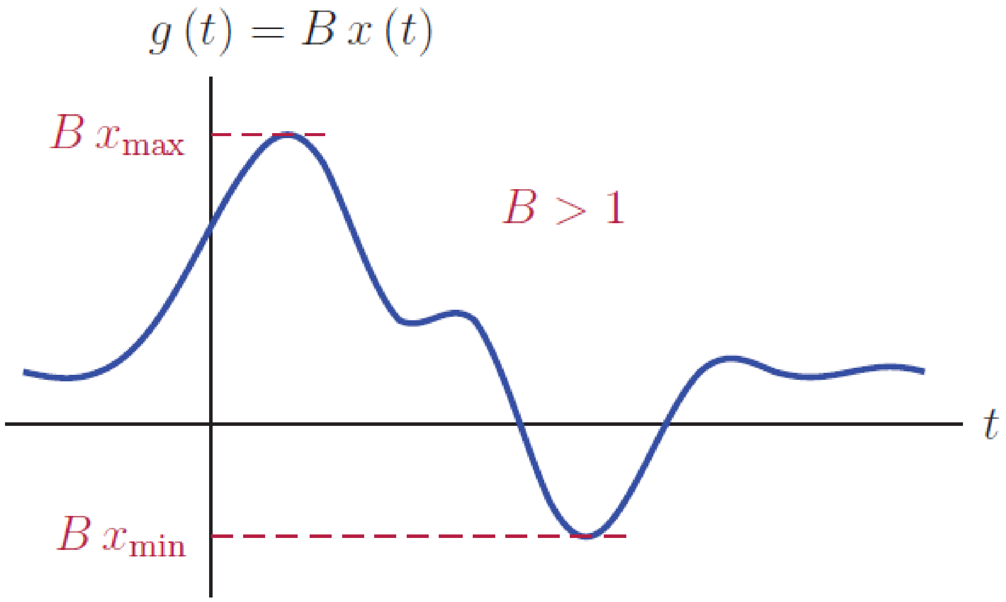


Signal Operations: Multiplication By a Constant Gain Factor

- A signal can also be multiplied with a constant gain factor

$$g(t) = Bx(t)$$





Signal Operations: Multiplication By a Constant Gain Factor

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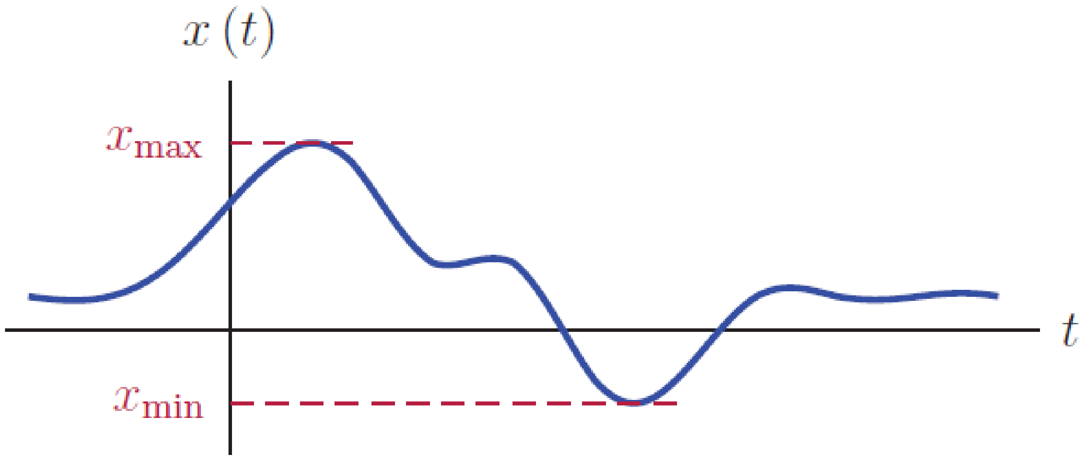
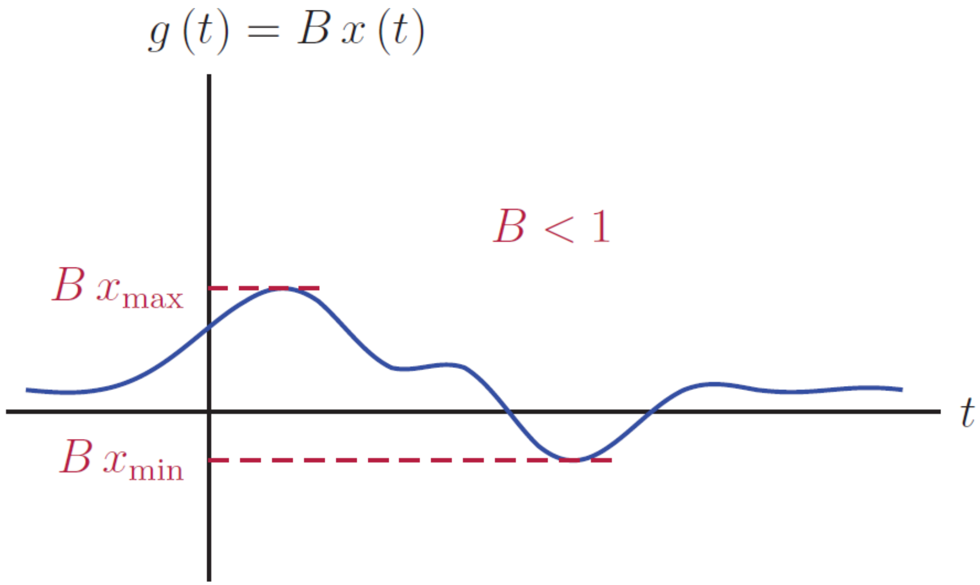
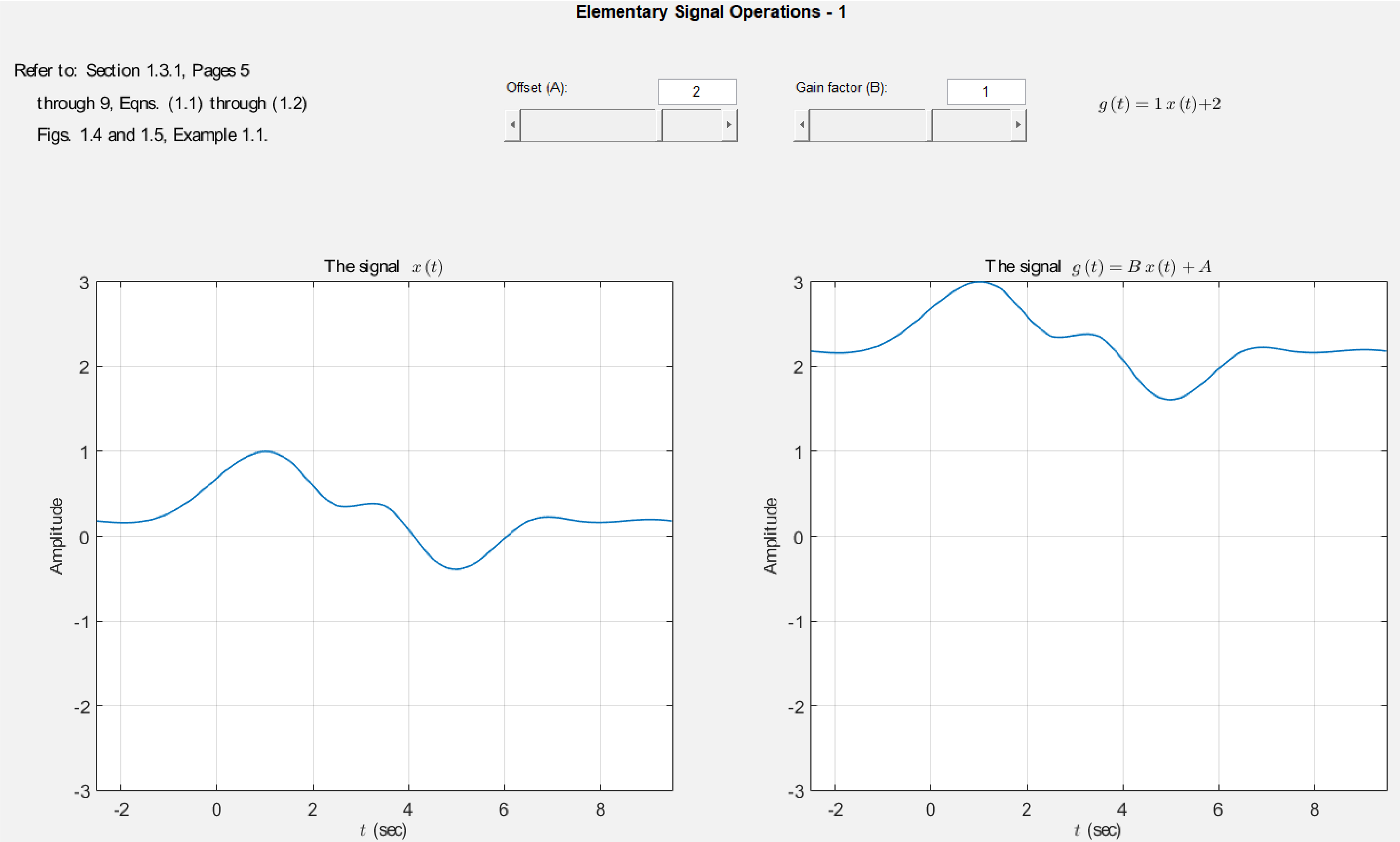
$$g(t) = Bx(t)$$

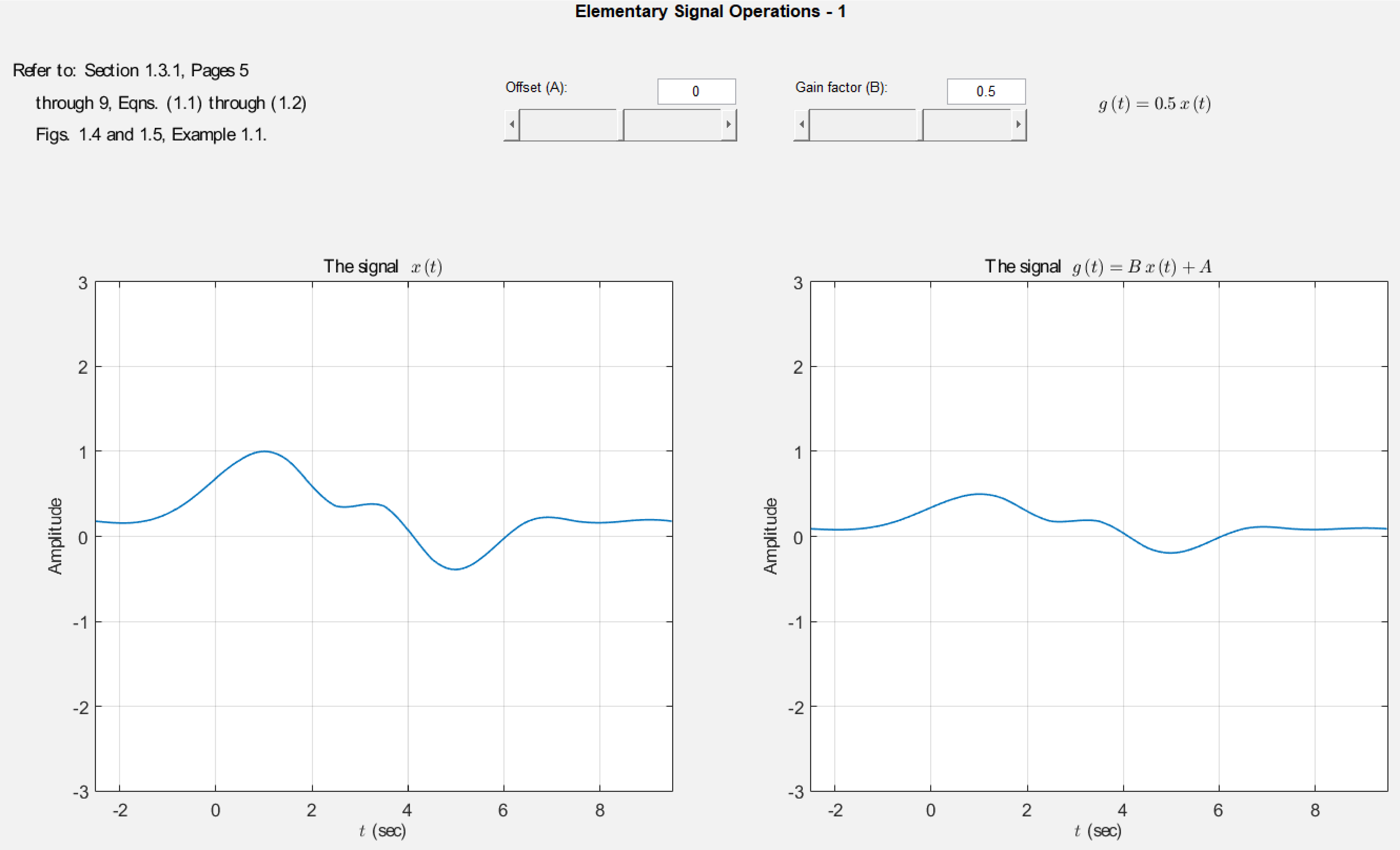




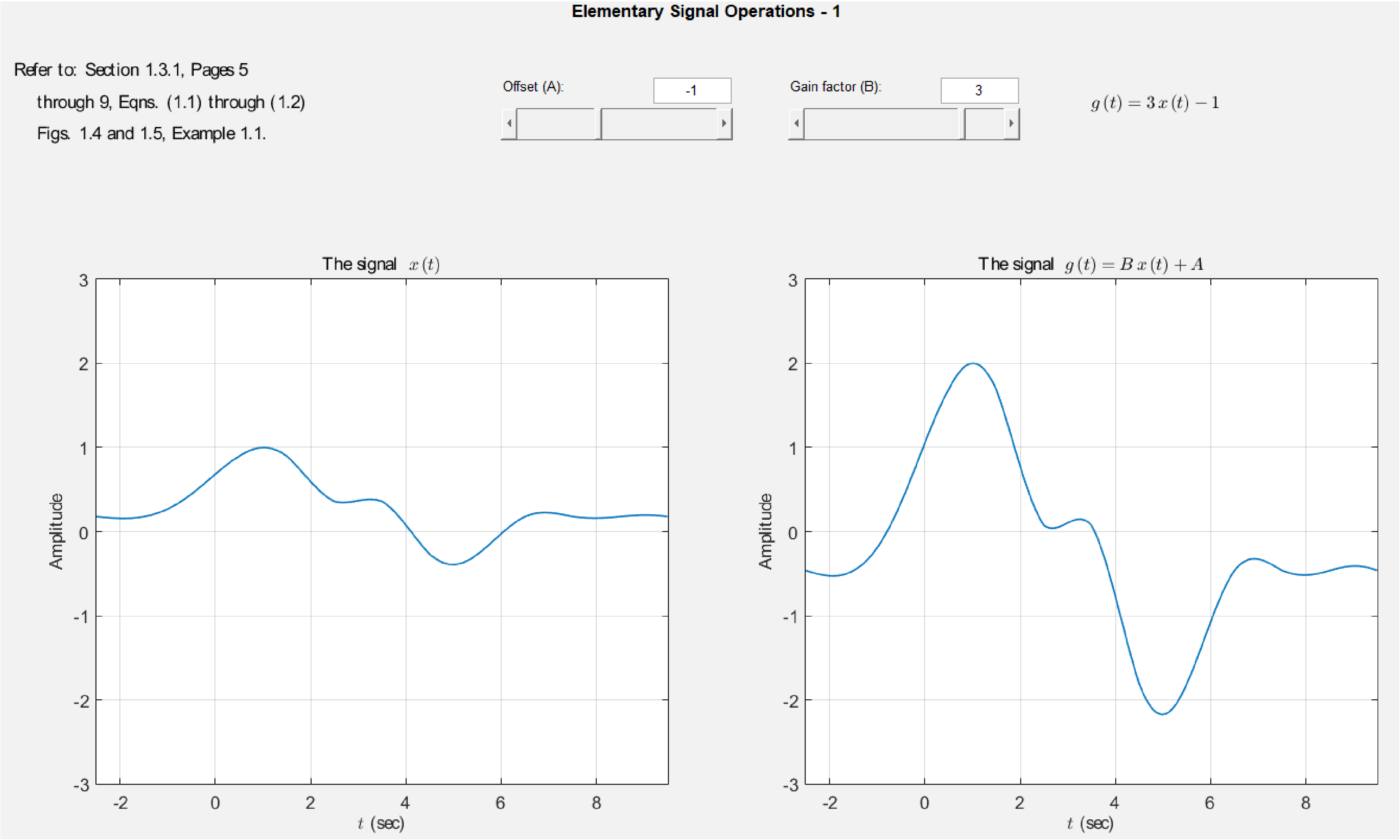
Signal Operations: sop_demo1



Signal Operations: sop_demo1



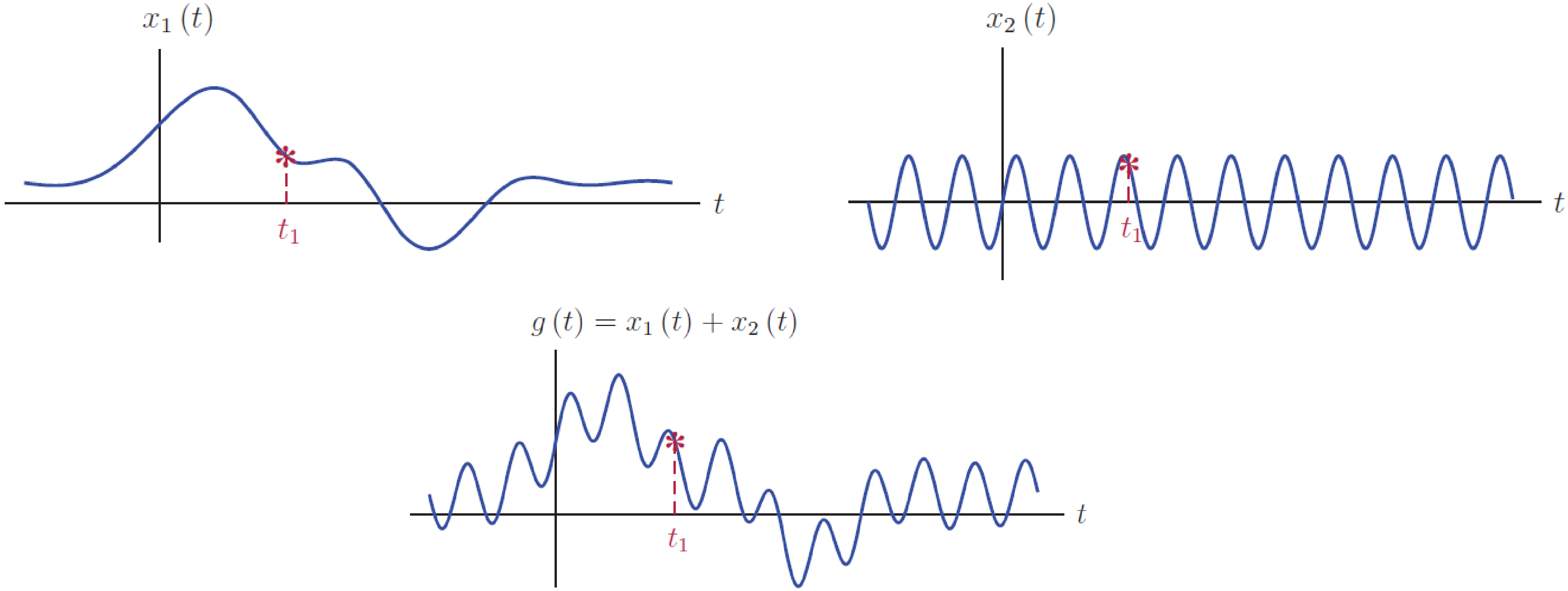
Signal Operations: sop_demo1



Signal Operations: Adding Signals

- Addition of two signals is accomplished by adding the amplitudes of the two signals at each time instant.

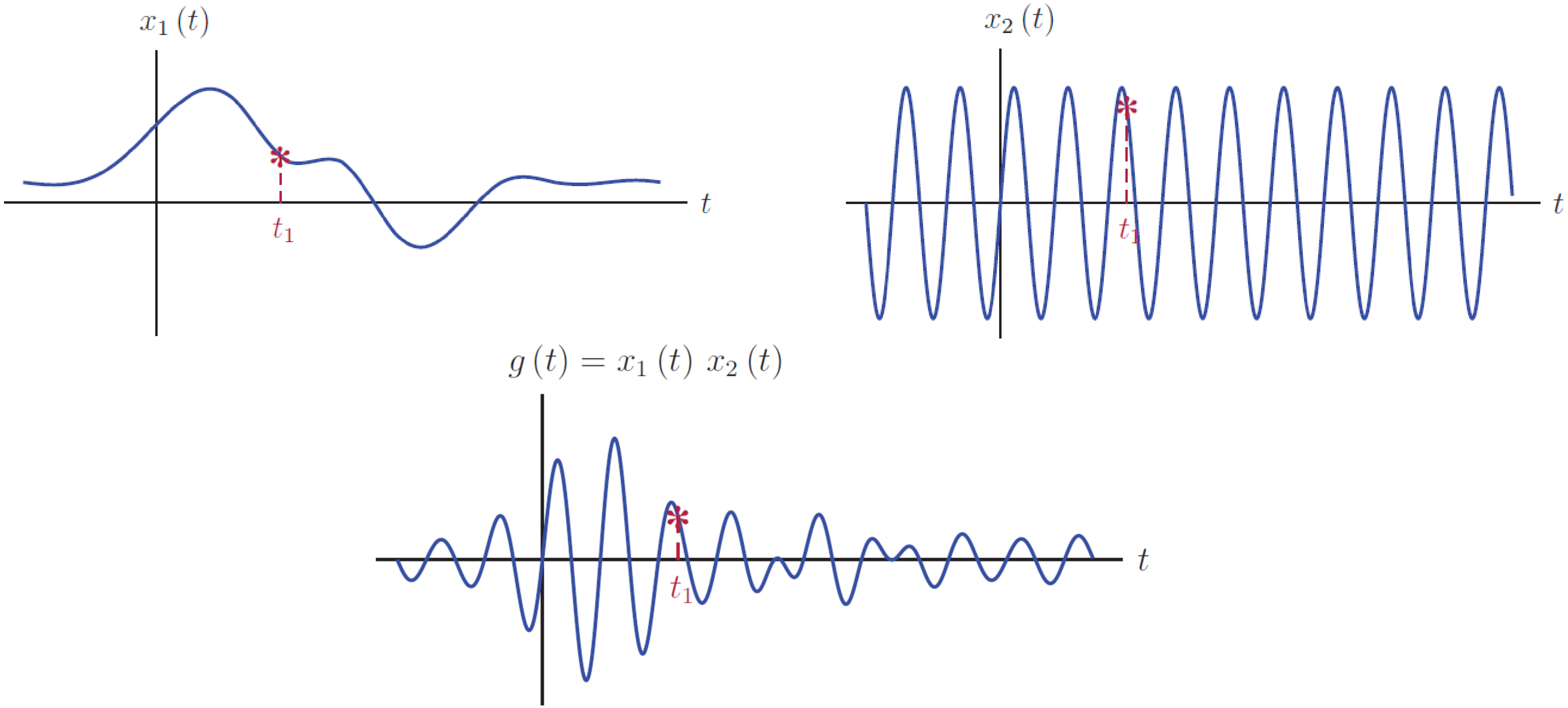
$$g(t) = x_1(t) + x_2(t)$$



Signal Operations: Multiplying Signals

- Multiplication of two signals is carried out in a similar manner.

$$g(t) = x_1(t)\, x_2(t)$$



Signal Operations: Example 1.2

Example 1.2: Arithmetic operations with continuous-time signals

Two signals $x_1(t)$ and $x_2(t)$ are shown in Fig. 1.11. Sketch the signals

a. $g_1(t) = x_1(t) + x_2(t)$

b. $g_2(t) = x_1(t)\, x_2(t)$

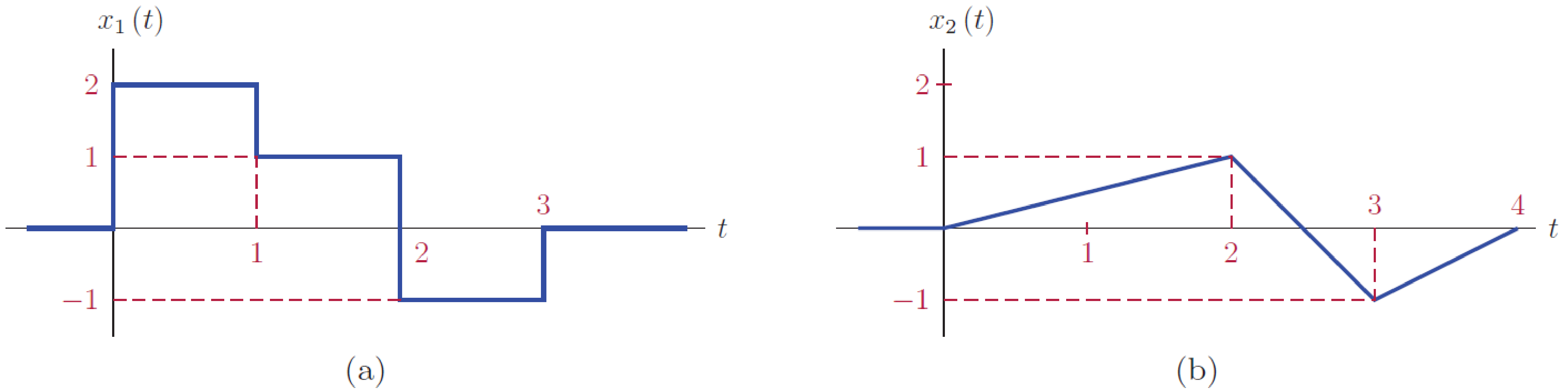
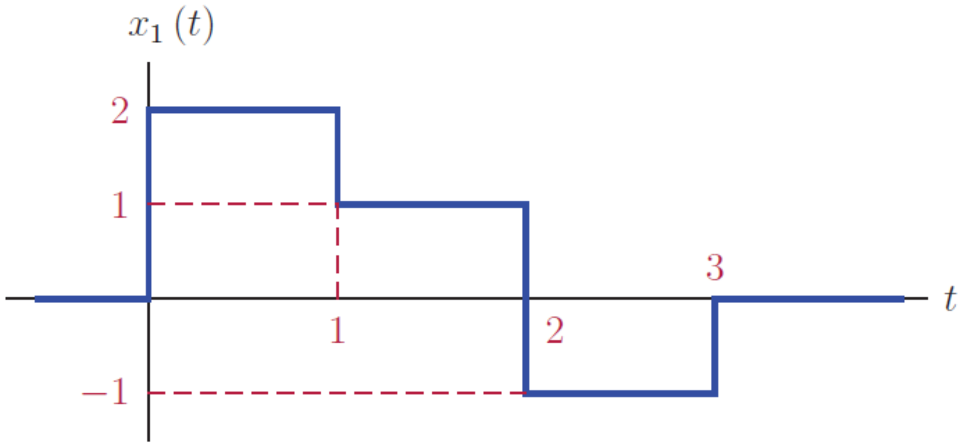
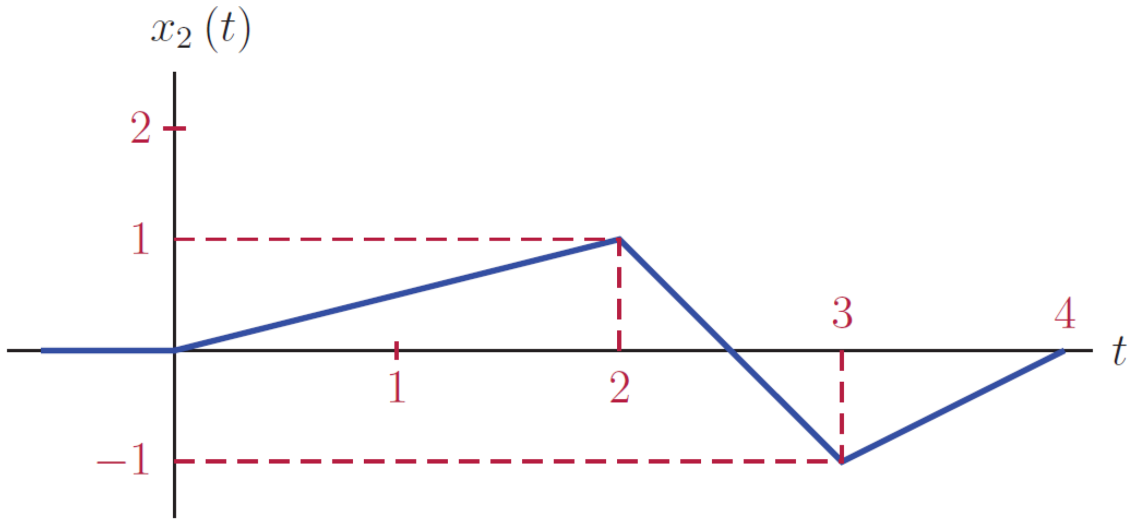


Figure 1.11 – Signals $x_1(t)$ and $x_2(t)$ for Example 1.2.

Signal Operations: Example 1.2 – Solution





$$x_1(t) = \begin{cases} 2\,, & 0 < t < 1 \\ 1\,, & 1 < t < 2 \\ -1\,, & 2 < t < 3 \\ 0\,, & \text{otherwise} \end{cases}$$

$$x_2(t) = \begin{cases} \frac{1}{2}t\,, & 0 < t < 2 \\ -2t + 5\,, & 2 < t < 3 \\ t - 4\,, & 3 < t < 4 \\ 0\,, & \text{otherwise} \end{cases}$$

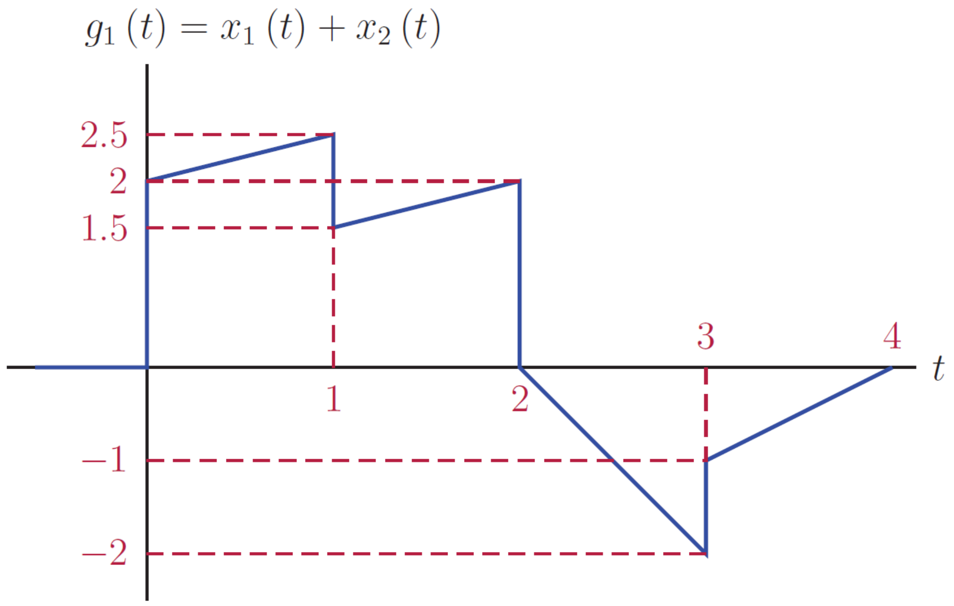
Signal Operations: Example 1.2 – Solution

$$x_1(t) = \begin{cases} 2\,, & 0 < t < 1 \\ 1\,, & 1 < t < 2 \\ -1\,, & 2 < t < 3 \\ 0\,, & \text{otherwise} \end{cases}$$

$$x_2(t) = \begin{cases} \frac{1}{2}t\,, & 0 < t < 2 \\ -2t+5\,, & 2 < t < 3 \\ t-4\,, & 3 < t < 4 \\ 0\,, & \text{otherwise} \end{cases}$$

The addition of the two signals is obtained as:

$$g_1(t) = \begin{cases} \frac{1}{2}t+2\,, & 0 < t < 1 \\ \frac{1}{2}t+1\,, & 1 < t < 2 \\ -2t+4\,, & 2 < t < 3 \\ t-4\,, & 3 < t < 4 \\ 0\,, & \text{otherwise} \end{cases}$$



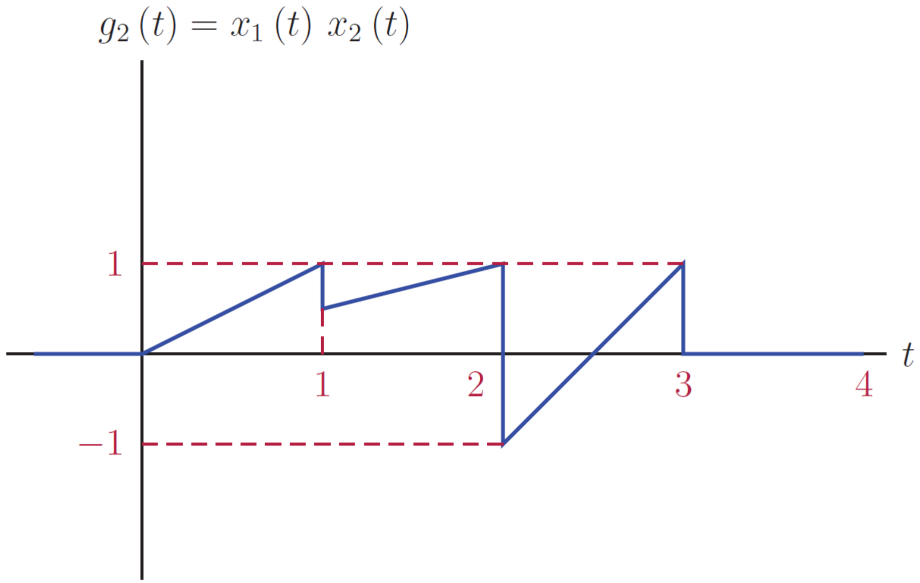
Signal Operations: Example 1.2 – Solution

$$x_1(t) = \begin{cases} 2\,, & 0 < t < 1 \\ 1\,, & 1 < t < 2 \\ -1\,, & 2 < t < 3 \\ 0\,, & \text{otherwise} \end{cases}$$

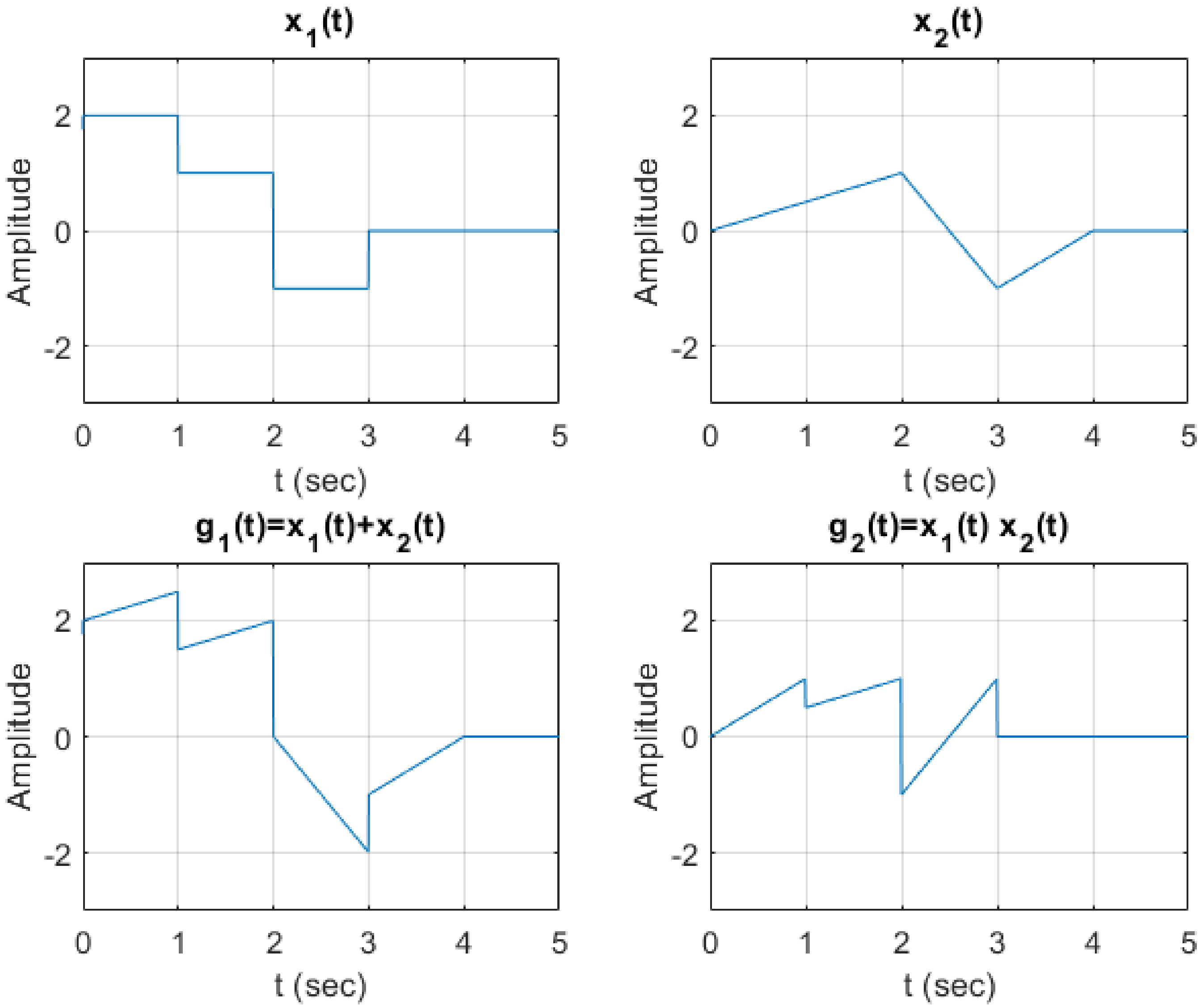
$$x_2(t) = \begin{cases} \frac{1}{2}t\,, & 0 < t < 2 \\ -2t+5\,, & 2 < t < 3 \\ t-4\,, & 3 < t < 4 \\ 0\,, & \text{otherwise} \end{cases}$$

The product of the two signals is obtained as:

$$g_2(t) = \begin{cases} t\,, & 0 < t < 1 \\ \frac{1}{2}t\,, & 1 < t < 2 \\ 2t-5\,, & 2 < t < 3 \\ 0\,, & \text{otherwise} \end{cases}$$



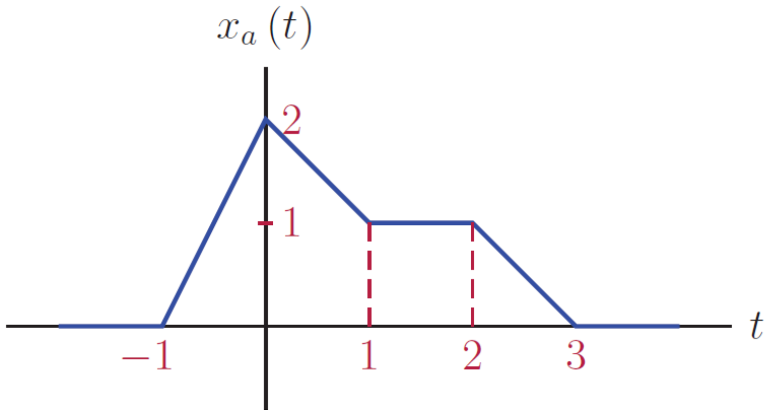
Signal Operations: Example 1.2 – MATLAB

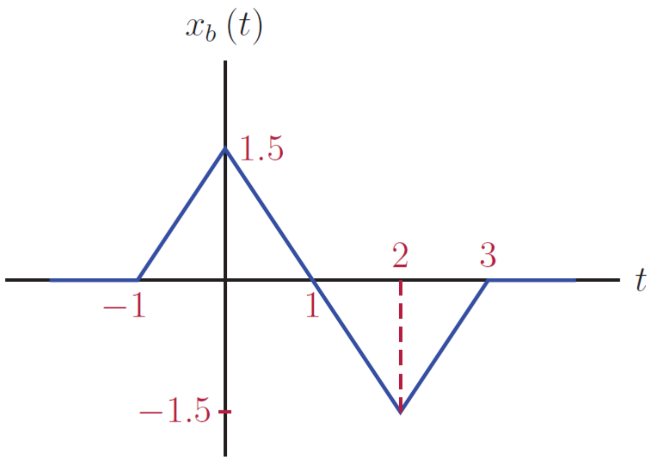


Signal Operations: Problem 1.3 (c)

1.3. Using the two signals $x_a(t)$ and $x_b(t)$ given in Fig. P.1.2, compute and sketch the signals specified below:

$$g_3(t) = 2x_a(t) - x_b(t) + 3$$





$$x_a(t) = \begin{cases} 0, & t < -1 \text{ or } t > 3 \\ 2t+2, & -1 < t < 0 \\ -t+2, & 0 < t < 1 \\ 1, & 1 < t < 2 \\ -t+3, & 2 < t < 3 \end{cases}$$

$$x_b(t) = \begin{cases} 0, & t < -1 \text{ or } t > 3 \\ 1.5t+1.5, & -1 < t < 0 \\ -1.5t+1.5, & 0 < t < 2 \\ 1.5t-4.5, & 2 < t < 3 \end{cases}$$

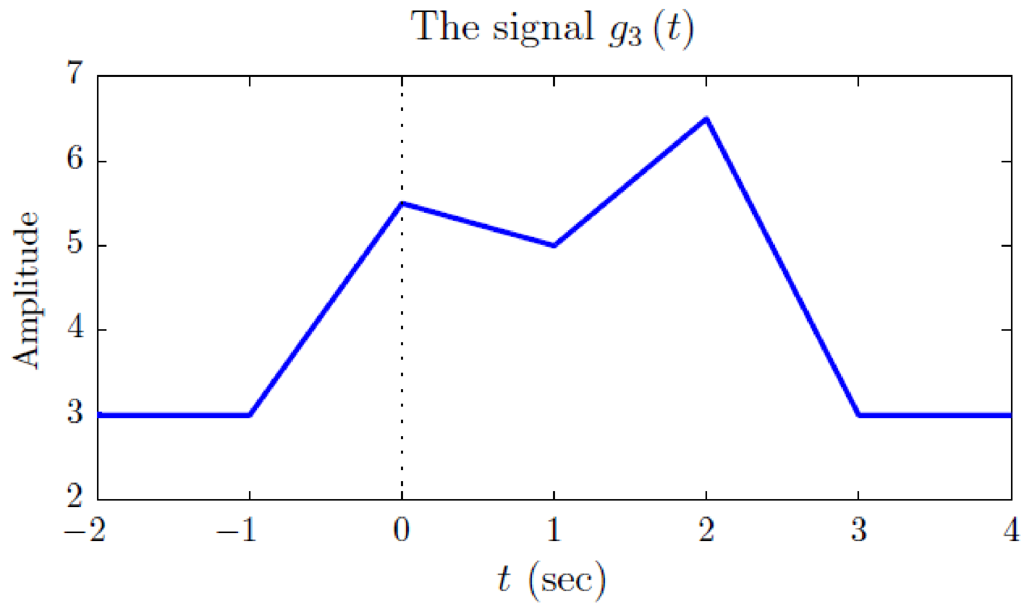
Signal Operations: Problem 1.3 (c) – Solution

$$x_a(t) = \begin{cases} 0\ , & t < -1 \text{ or } t > 3 \\ 2t+2\ , & -1 < t < 0 \\ -t+2\ , & 0 < t < 1 \\ 1\ , & 1 < t < 2 \\ -t+3\ , & 2 < t < 3 \end{cases}$$

$$x_b(t) = \begin{cases} 0\ , & t < -1 \text{ or } t > 3 \\ 1.5t+1.5\ , & -1 < t < 0 \\ -1.5t+1.5\ , & 0 < t < 2 \\ 1.5t-4.5\ , & 2 < t < 3 \end{cases}$$

$$g_3(t) = 2x_a(t) - x_b(t) + 3$$

$$g_3(t) = \begin{cases} 3\ , & t < -1 \text{ or } t > 3 \\ 2.5t+5.5\ , & -1 < t < 0 \\ -0.5t+5.5\ , & 0 < t < 1 \\ 1.5t+3.5\ , & 1 < t < 2 \\ -3.5t+13.5\ , & 2 < t < 3 \end{cases}$$

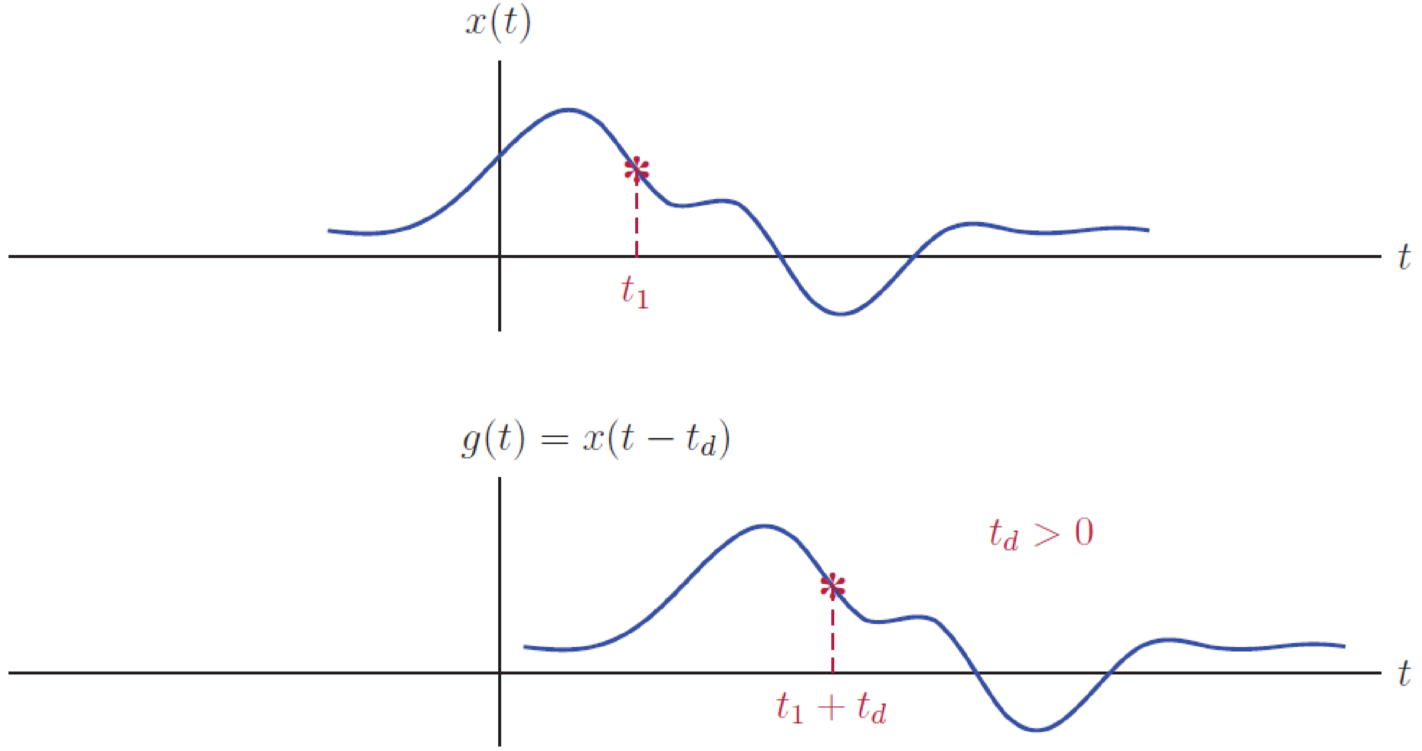


Signal Operations: Time Shifting

- A time shifted version of the signal $x(t)$ can be obtained through

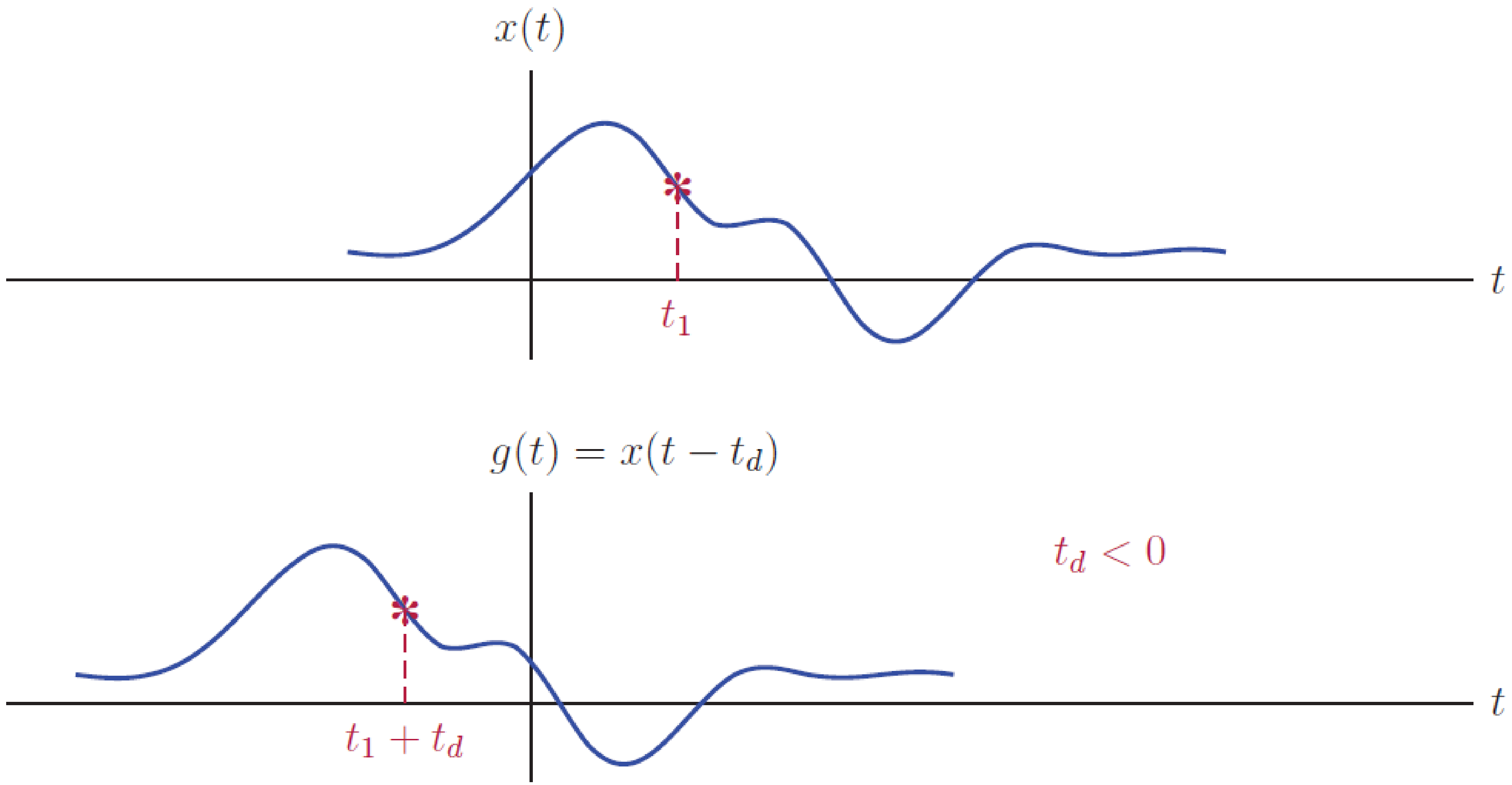
$$g(t) = x(t - t_d)$$

- If t_d is positive, $g(t)$ is a delayed version of $x(t)$.



Signal Operations: Time Shifting

- A negative t_d, on the other hand, corresponds to advancing the signal in time by an amount equal to $-t_d$.

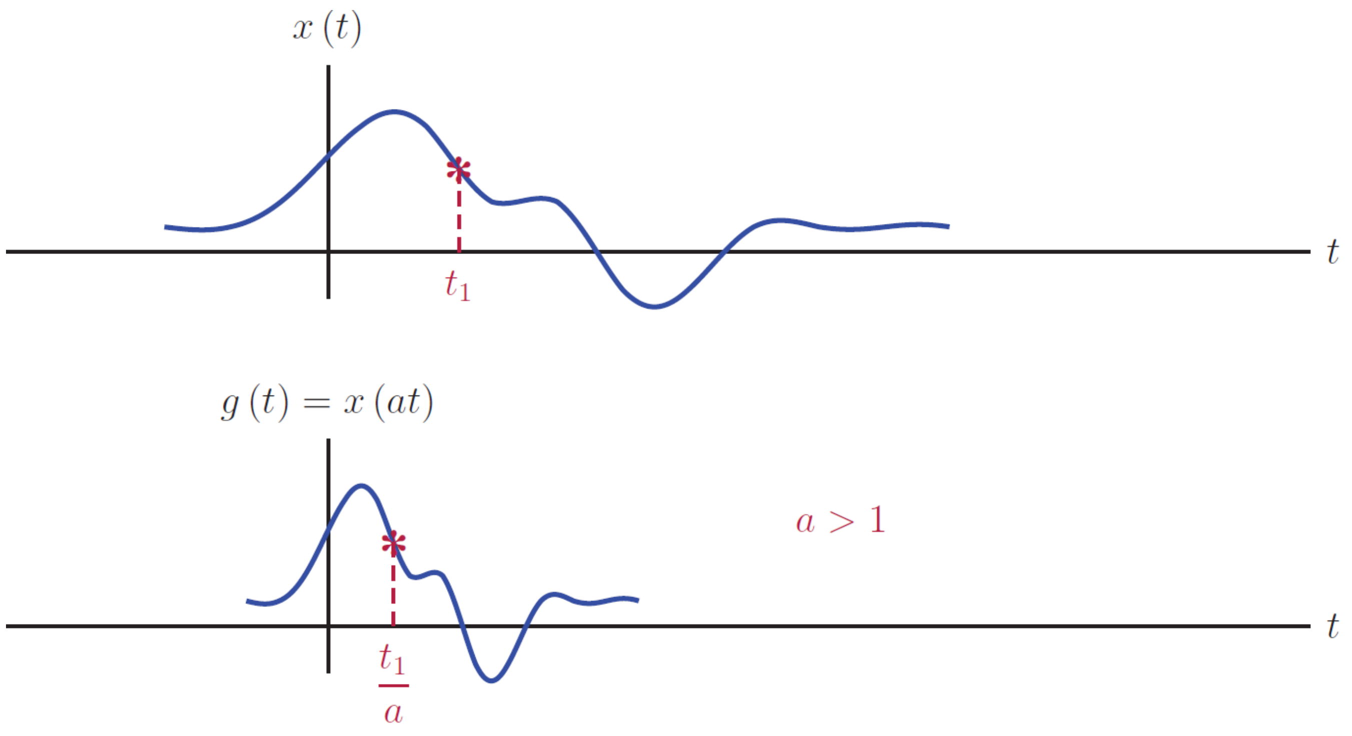


Signal Operations: Time Scaling

- A time scaled version of the signal $x(t)$ is obtained through

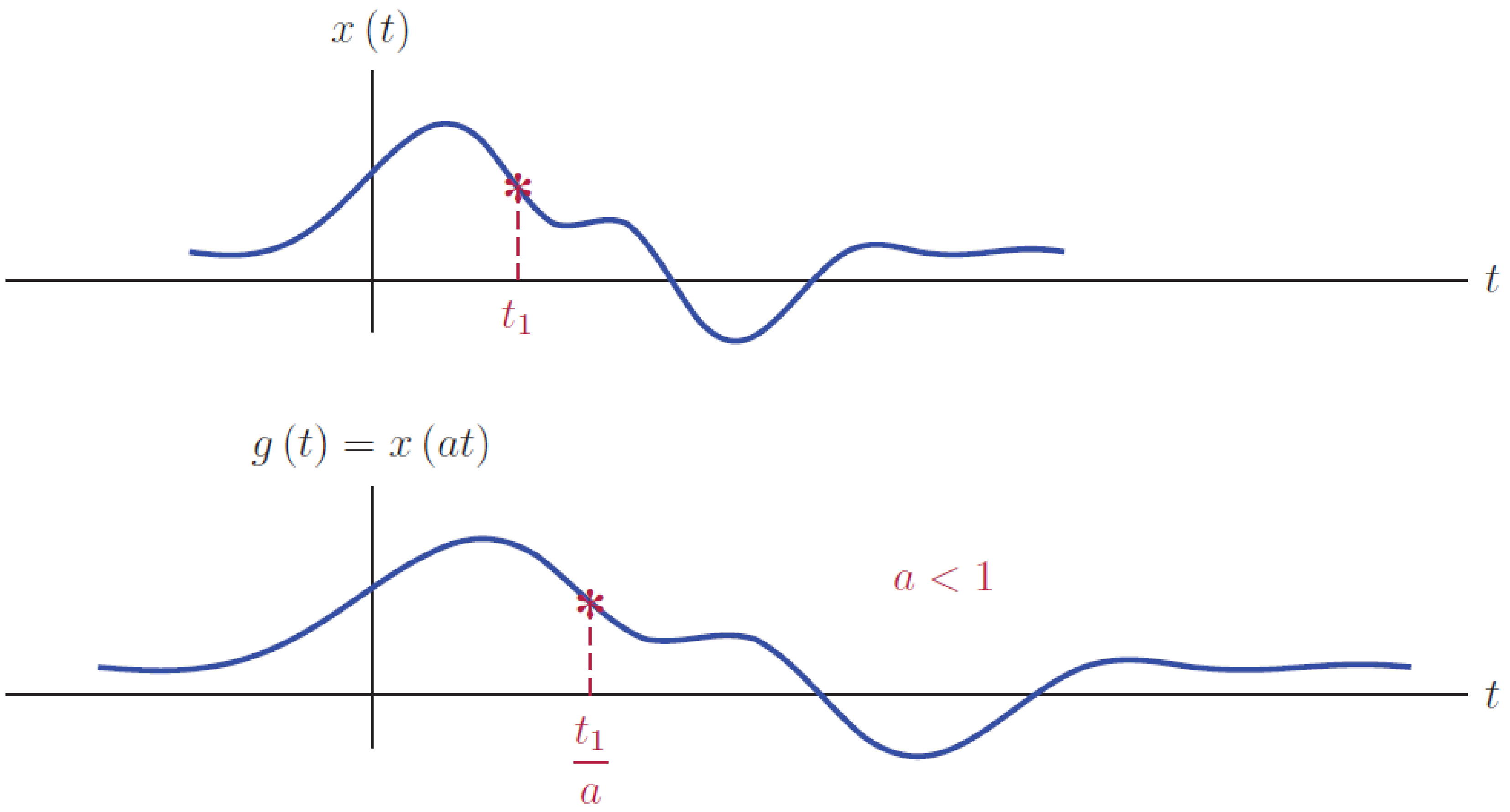
$$g(t) = x(at)$$

- A scaling parameter value of $a > 1$ results in the signal $g(t)$ being a compressed version of $x(t)$.



Signal Operations: Time Scaling

- Conversely, $a < 1$ leads to a signal $g(t)$ that is an expanded version of $x(t)$.

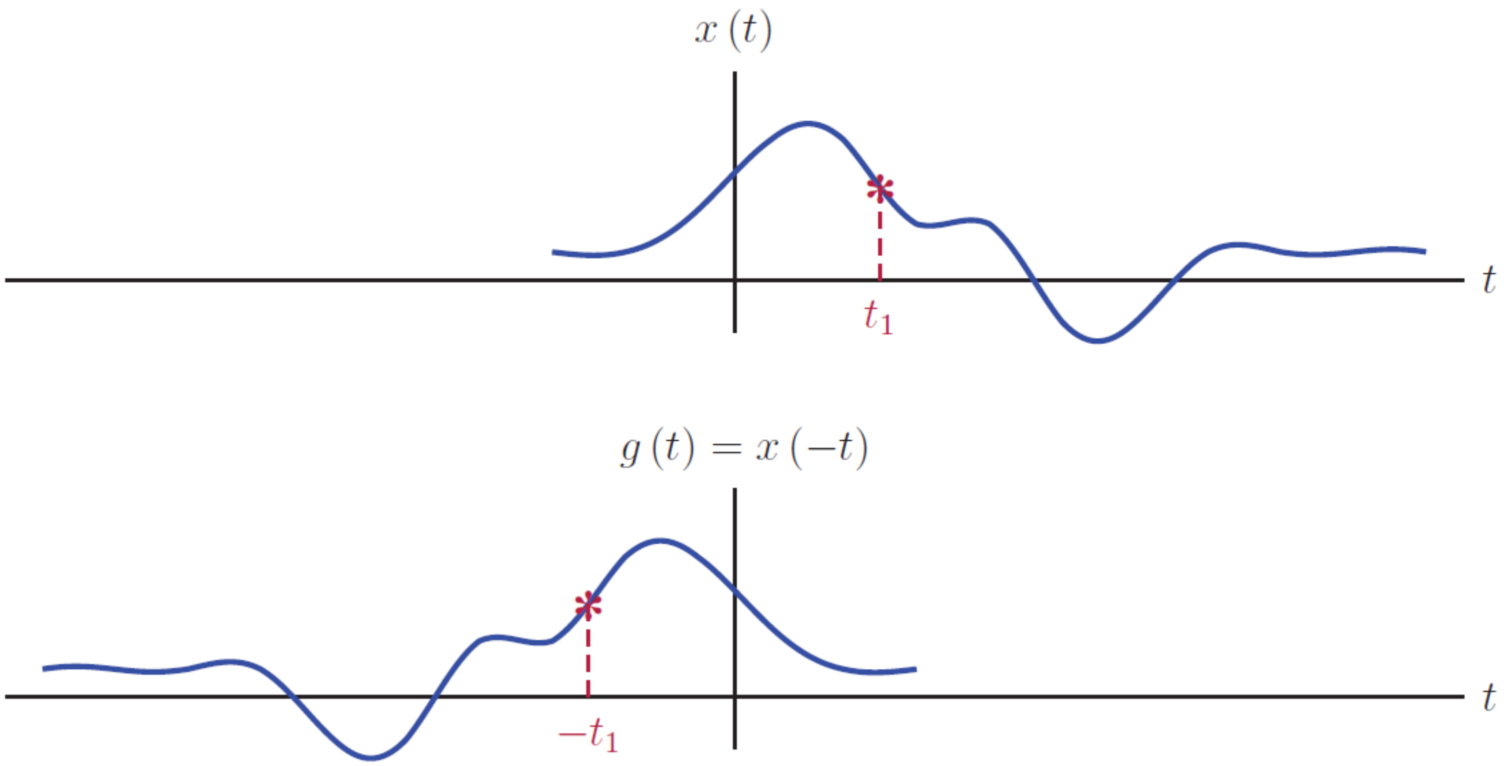


Signal Operations: Time Reversal

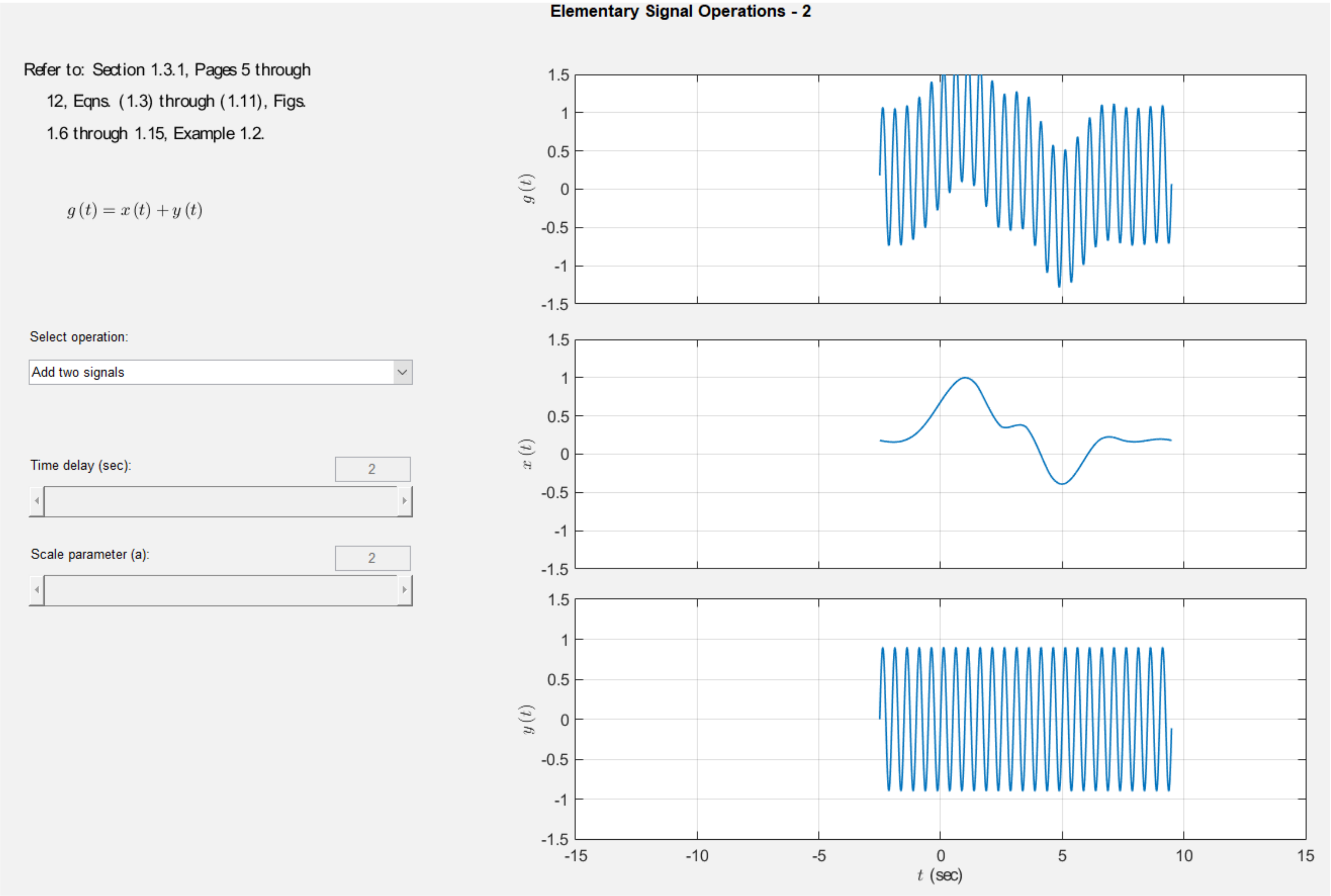
- A time reversed version of the signal $x(t)$ is obtained through

$$g(t) = x(-t)$$

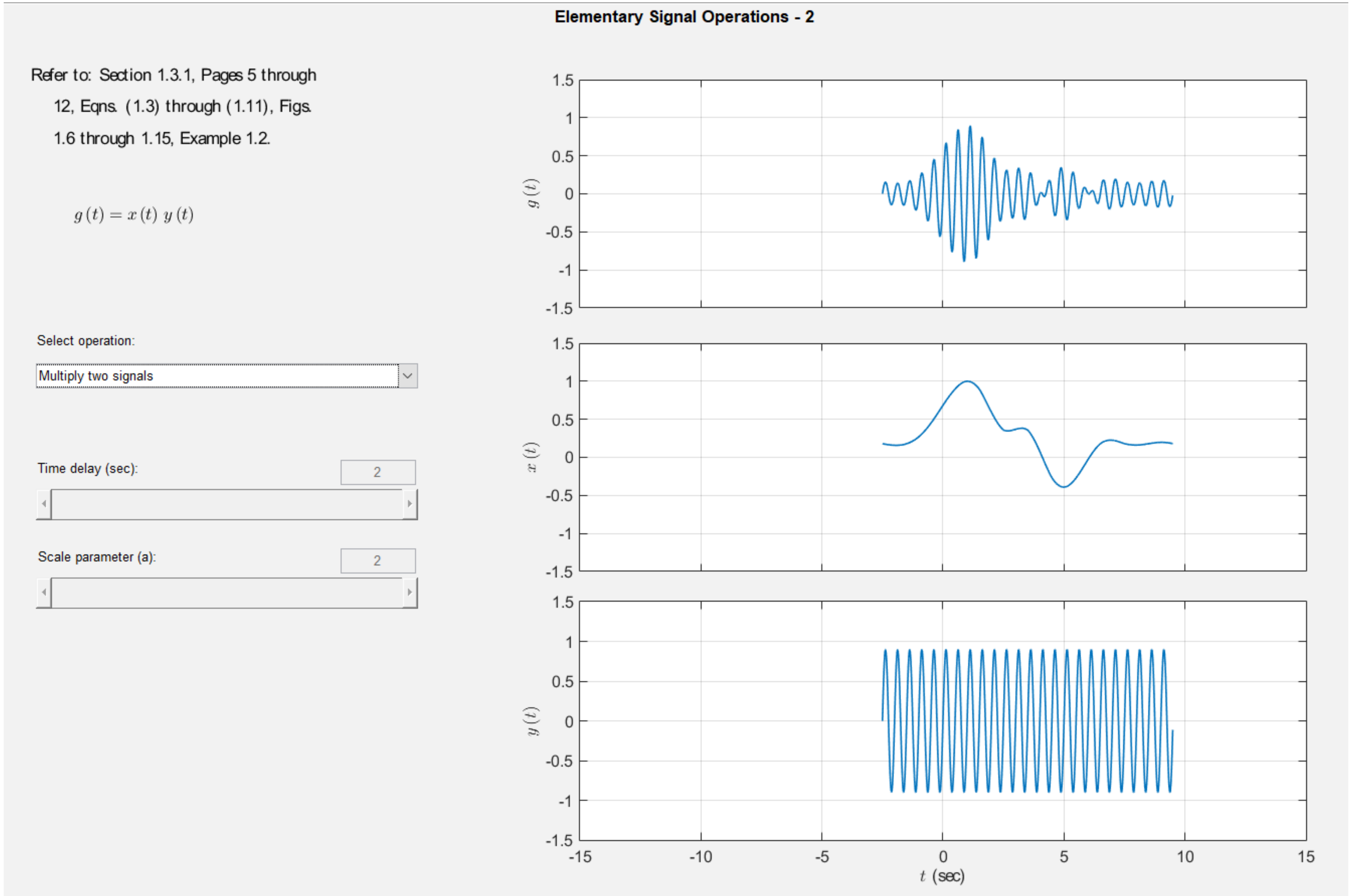
- Graphically this corresponds to flipping the signal around the vertical axis.



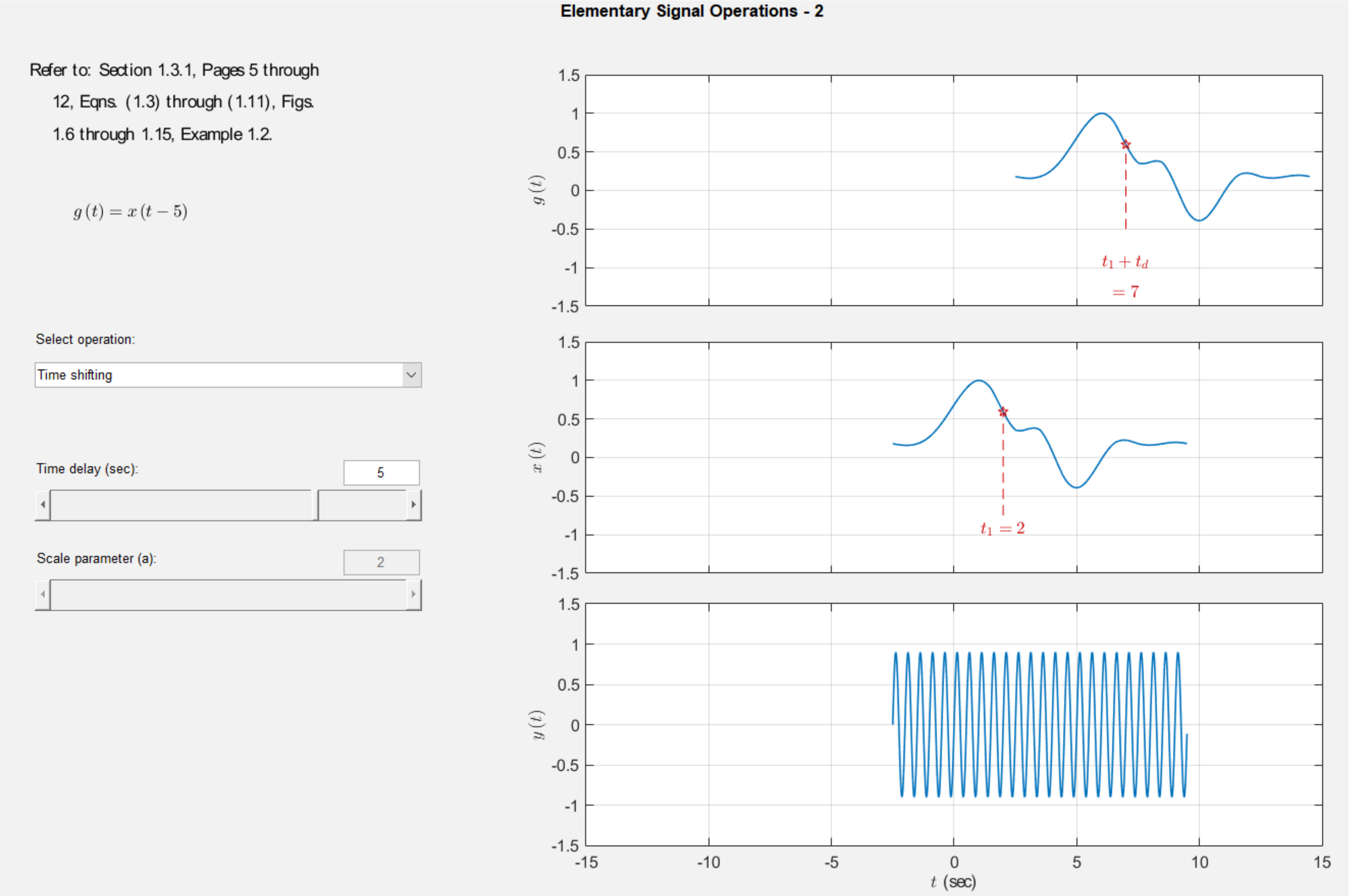
Signal Operations: sop_demo2



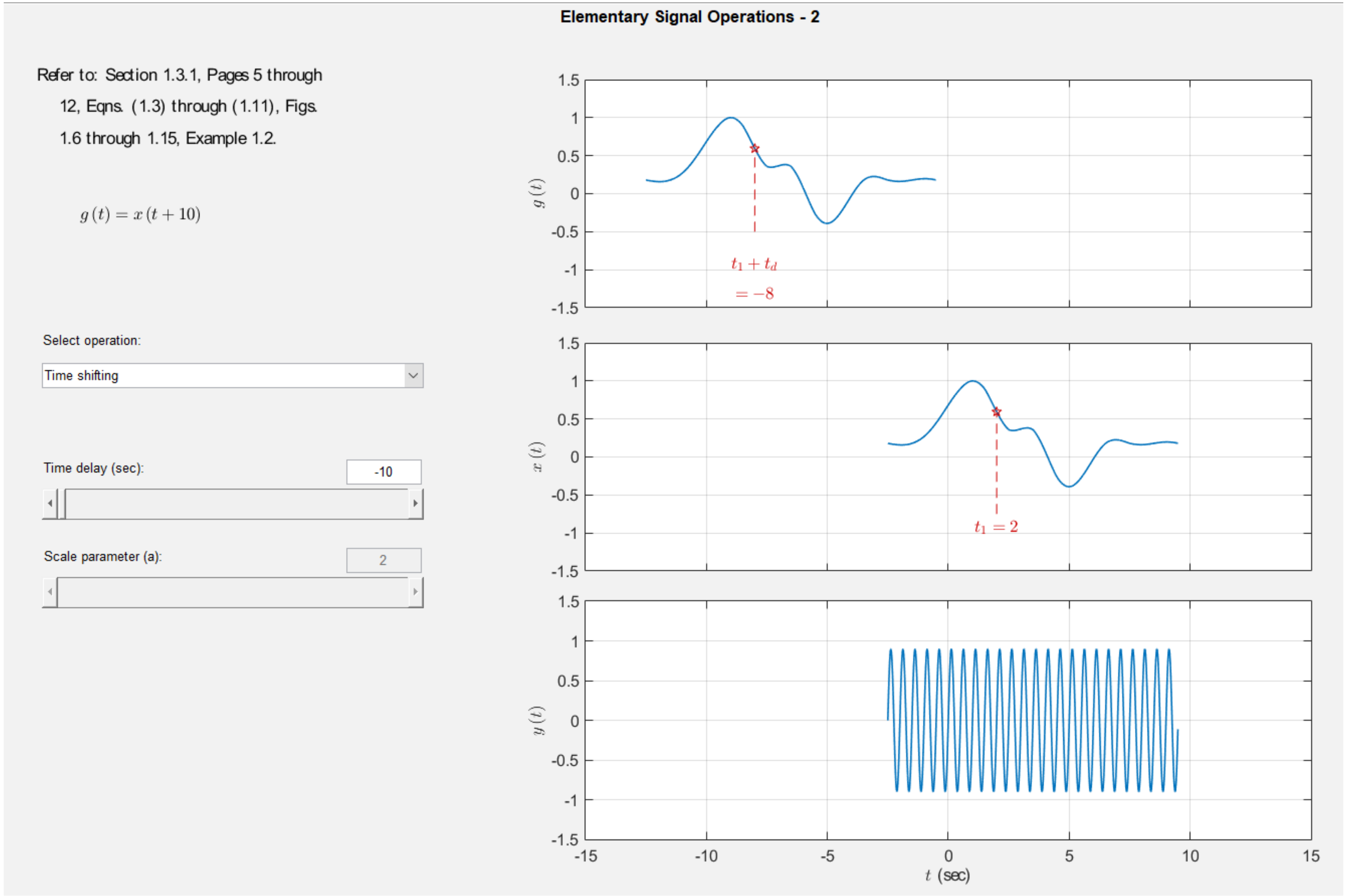
Signal Operations: sop_demo2



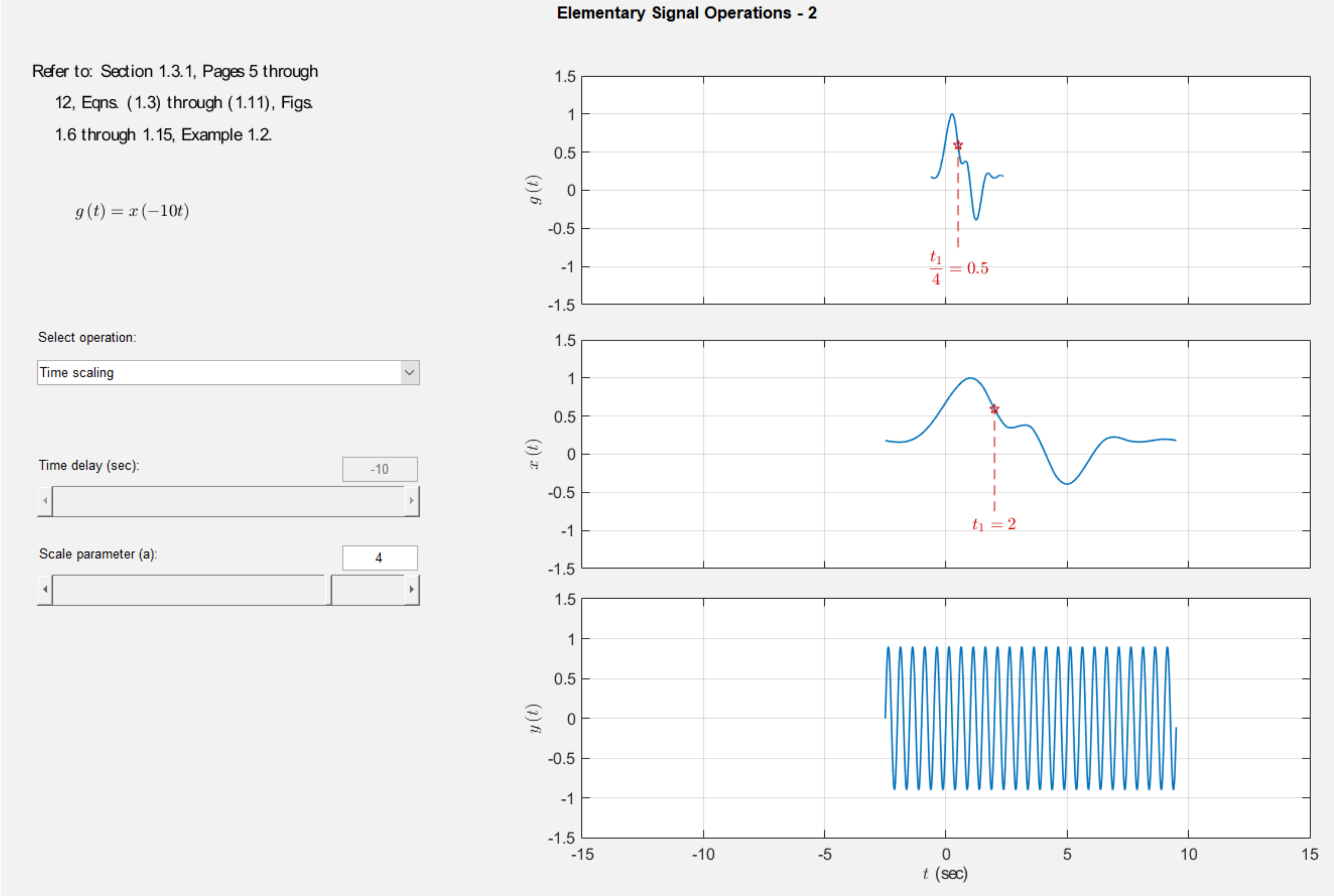
Signal Operations: sop_demo2



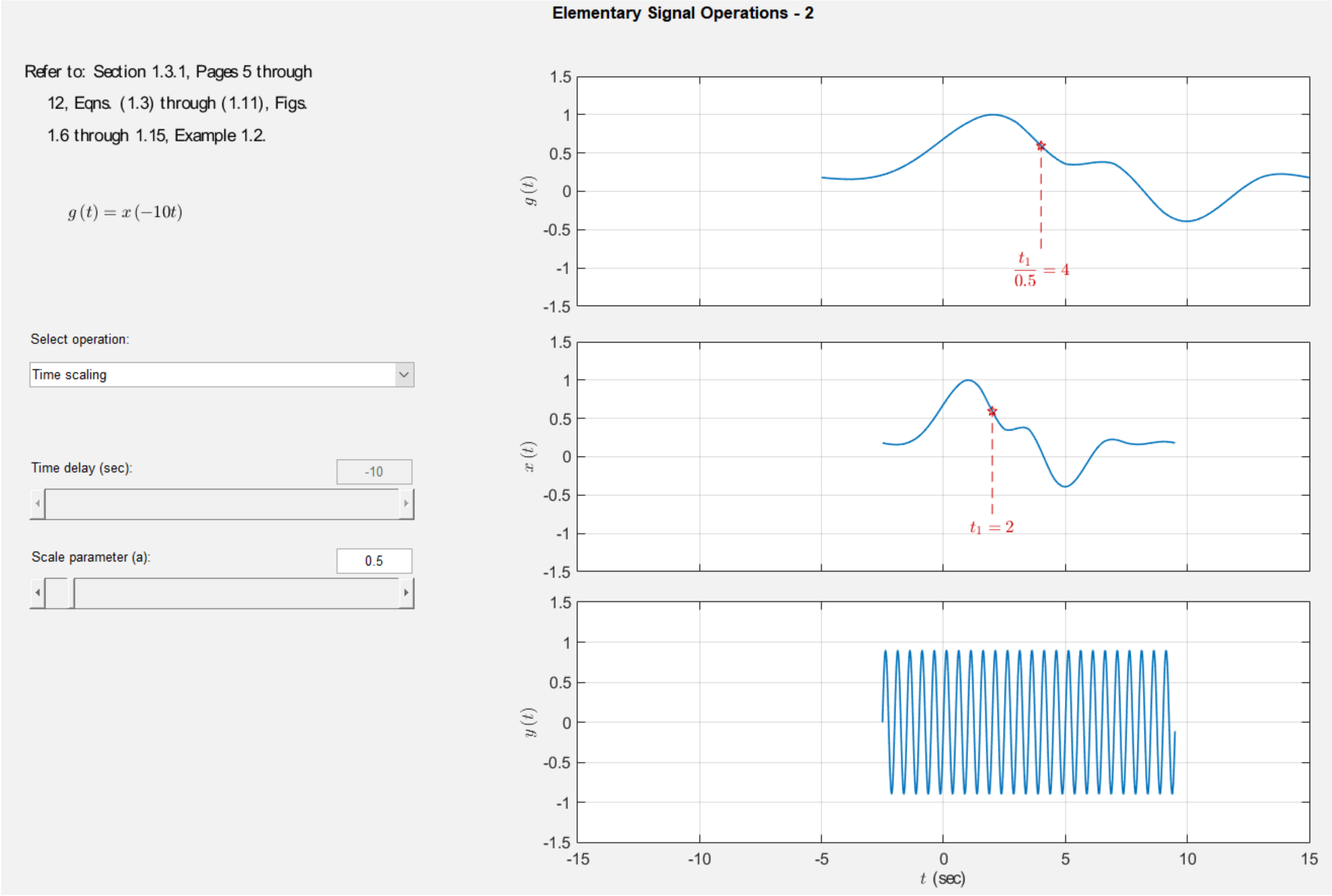
Signal Operations: sop_demo2



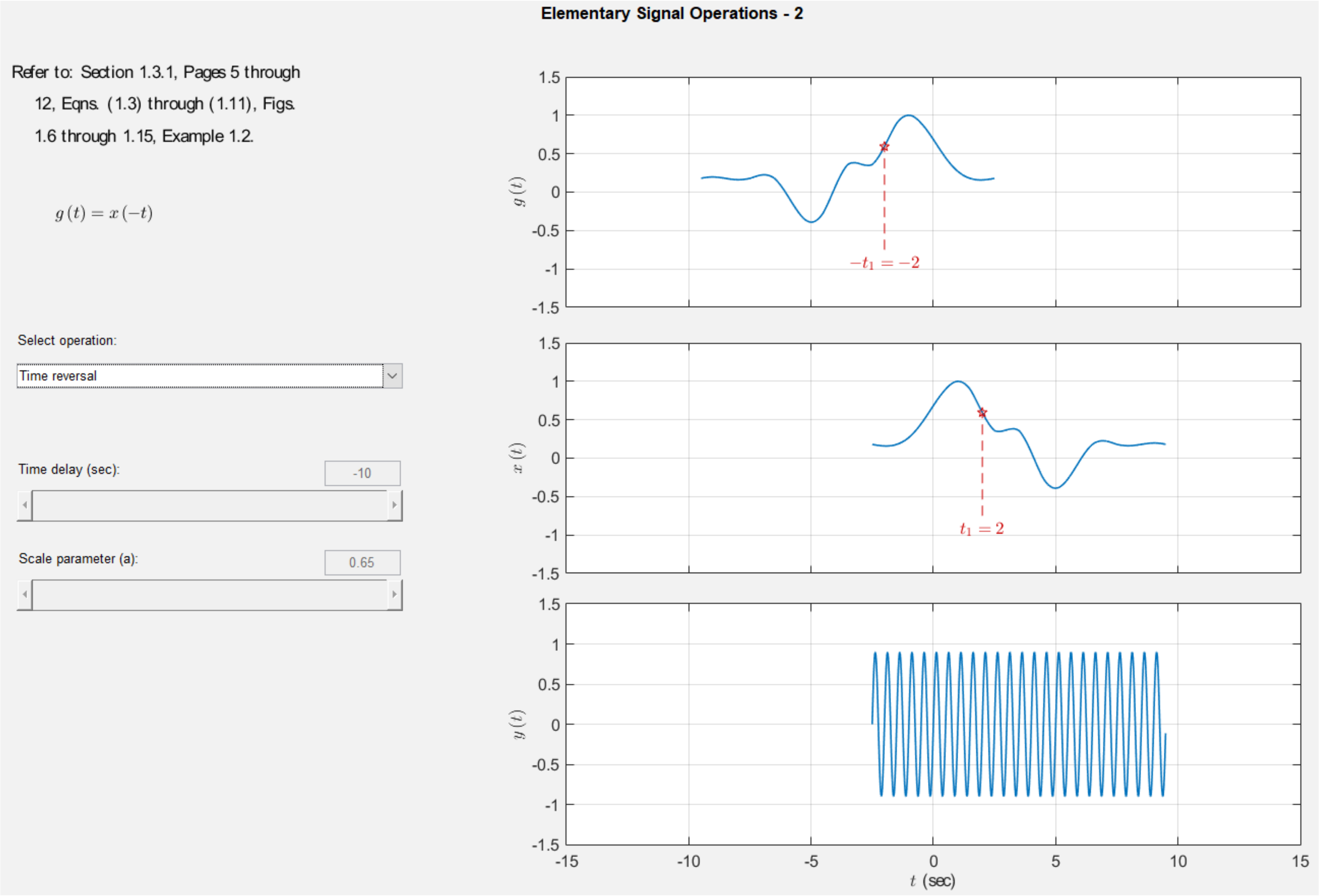
Signal Operations: sop_demo2



Signal Operations: sop_demo2



Signal Operations: sop_demo2



Signal Operations: Example 1.3

Example 1.3: Basic operations for continuous-time signals

Consider the signal $x(t)$ shown in Fig. 1.16. Sketch the following signals:

a. $g(t) = x(2t - 5)$,

b. $h(t) = x(-4t + 2)$.

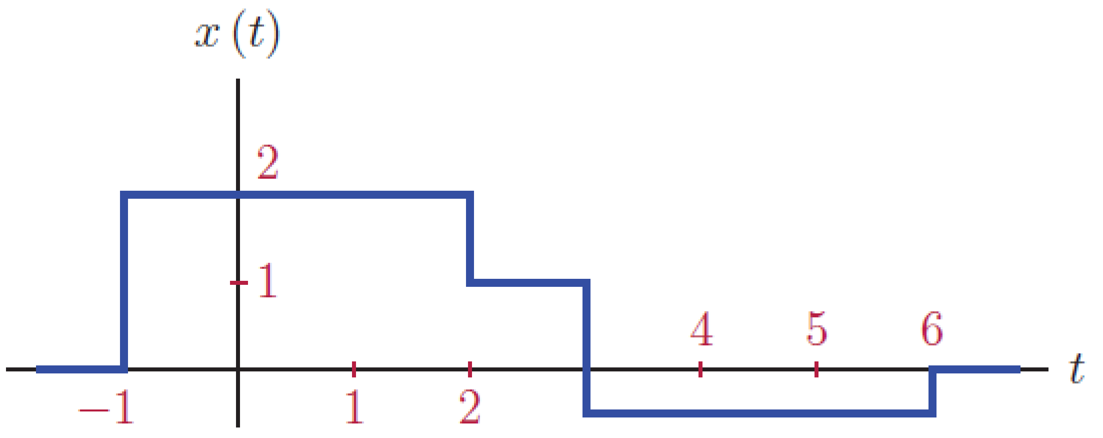
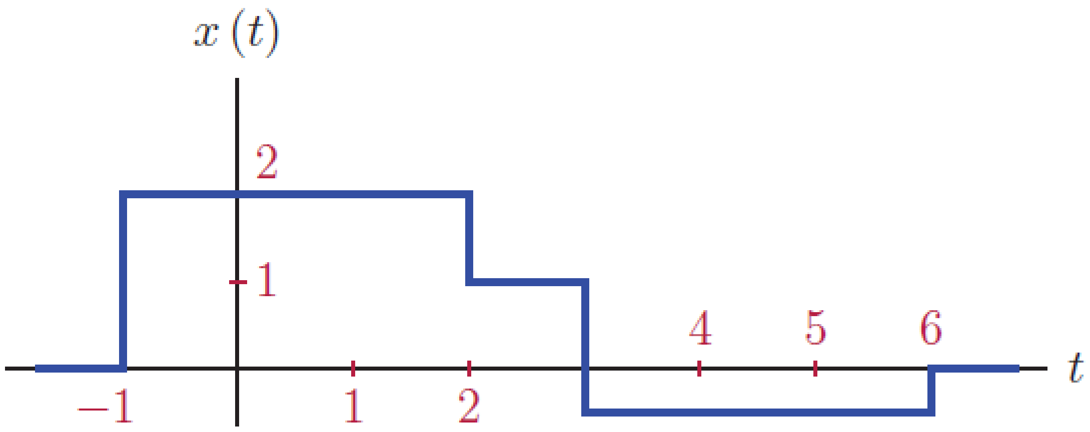


Figure 1.16 – The signal $x(t)$ for Example 1.3.

Signal Operations: Example 1.3 (a) – Solution



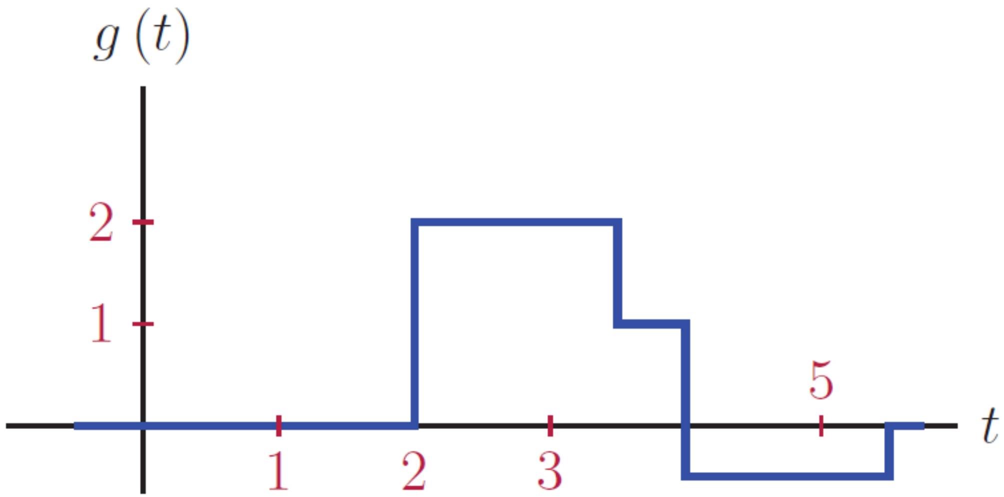
$$x(t) = \begin{cases} 2, & -1 < t < 2 \\ 1, & 2 < t < 3 \\ -0.5, & 3 < t < 6 \\ 0, & \text{otherwise} \end{cases}$$

$$x(2t-5) = \begin{cases} 2, & -1 < 2t-5 < 2 \\ 1, & 2 < 2t-5 < 3 \\ -0.5, & 3 < 2t-5 < 6 \\ 0, & \text{otherwise} \end{cases}$$

Signal Operations: Example 1.3 (a) – Solution

$$x(2t-5)=\begin{cases}2, & 4<2t<7\\ 1, & 7<2t<8\\ -0.5, & 8<2t<11\\ 0, & \text{otherwise}\end{cases}$$

$$x(2t-5)=\begin{cases}2, & 2<t<3.5\\ 1, & 3.5<t<4\\ -0.5, & 4<t<5.5\\ 0, & \text{otherwise}\end{cases}$$



Signal Operations: Example 1.3 (a) – Another Solution

Solution:

a. We will obtain $g(t)$ in two steps: Let an intermediate signal be defined as $g_1(t) = x(2t)$, a time scaled version of $x(t)$, shown in Fig. 1.17(b). The signal $g(t)$ can be expressed as

$$g(t) = g_1(t - 2.5) = x(2[t - 2.5]) = x(2t - 5)$$

and is shown in Fig. 1.17(c).

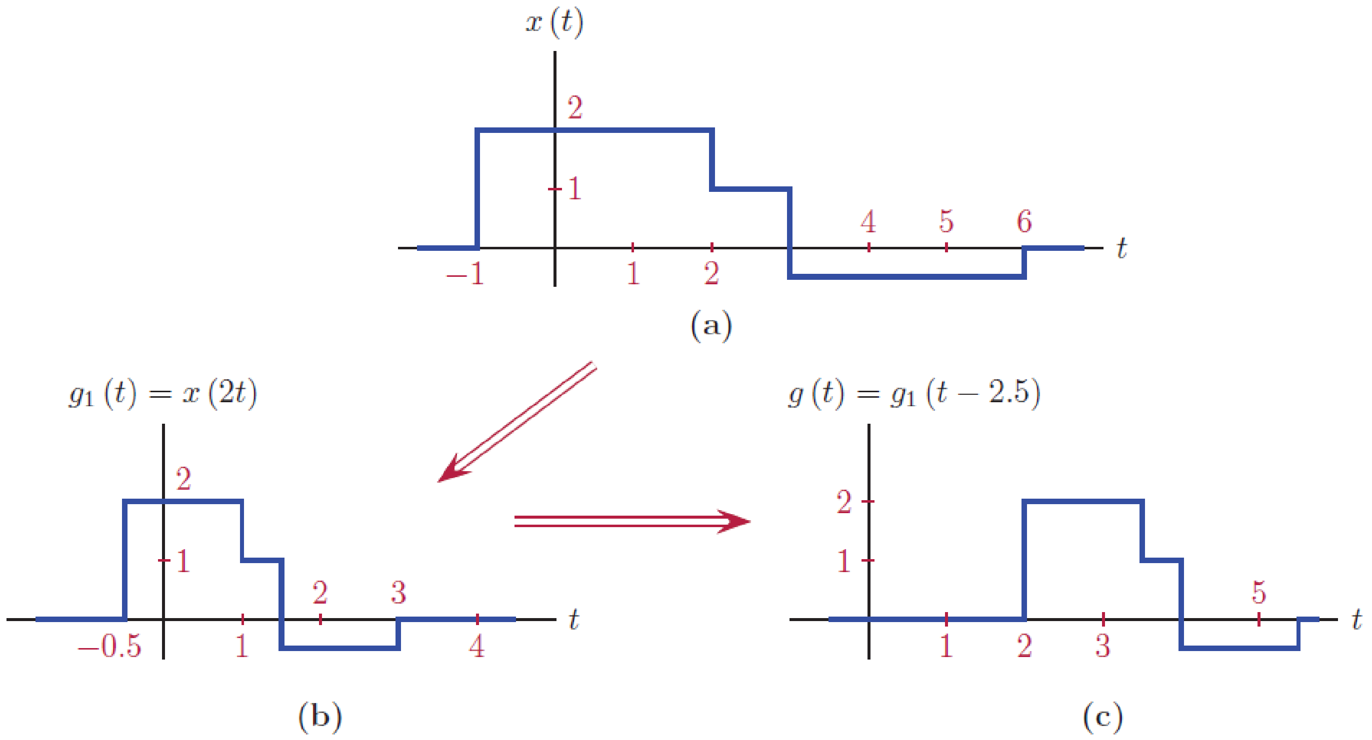
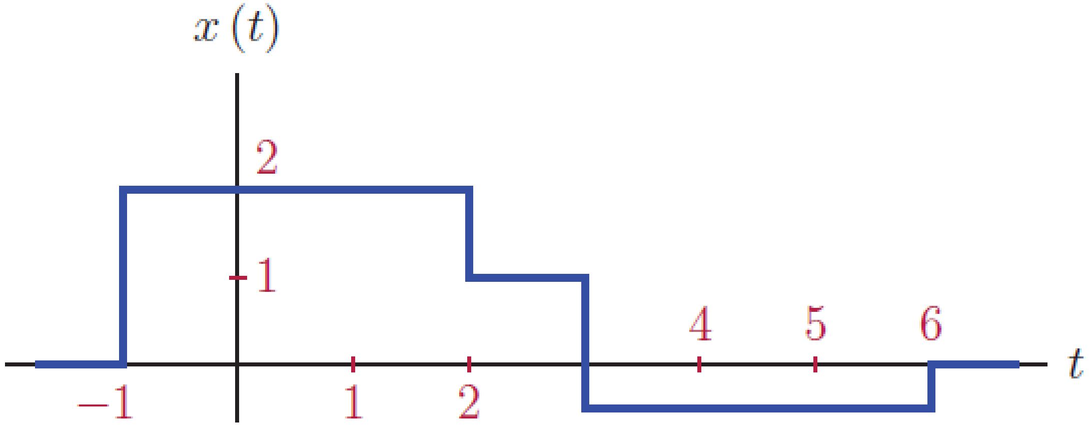


Figure 1.17 – (a) The intermediate signal $g_1(t)$, and (b) the signal $g(t)$ for Example 1.3.

Signal Operations: Example 1.3 (b) – Solution



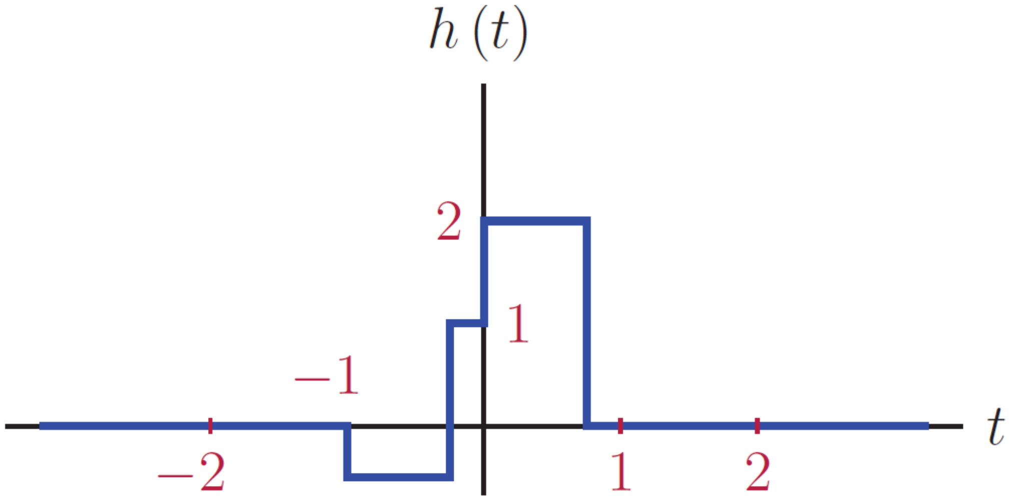
$$x(t) = \begin{cases} 2, & -1 < t < 2 \\ 1, & 2 < t < 3 \\ -0.5, & 3 < t < 6 \\ 0, & \text{otherwise} \end{cases}$$

$$x(-4t+2) = \begin{cases} 2, & -1 < -4t+2 < 2 \\ 1, & 2 < -4t+2 < 3 \\ -0.5, & 3 < -4t+2 < 6 \\ 0, & \text{otherwise} \end{cases}$$

Signal Operations: Example 1.3 (b) – Solution

$$x(-4t+2)=\begin{cases}2, & -3<-4t<0\\ 1, & 0<-4t<1\\ -0.5, & 1<-4t<4\\ 0, & \text{otherwise}\end{cases}$$

$$x(-4t+2)=\begin{cases}2, & 0.75>t>0\\ 1, & 0>t>-0.25\\ -0.5, & -0.25>t>-1\\ 0, & \text{otherwise}\end{cases}$$



Signal Operations: Example 1.3 (b) – Another Solution

b. In this case we will use two intermediate signals: Let $h_1(t) = x(4t)$. A second intermediate signal $h_2(t)$ can be obtained by time shifting $h_1(t)$ so that

$$h_2(t) = h_1(t + 0.5) = x(4[t + 0.5]) = x(4t + 2)$$

Finally, $h(t)$ can be obtained through time reversal of $h_2(t)$:

$$h(t) = h_2(-t) = x(-4t + 2)$$

The steps involved in sketching $h(t)$ are shown in Fig. 1.18(a)–(d).

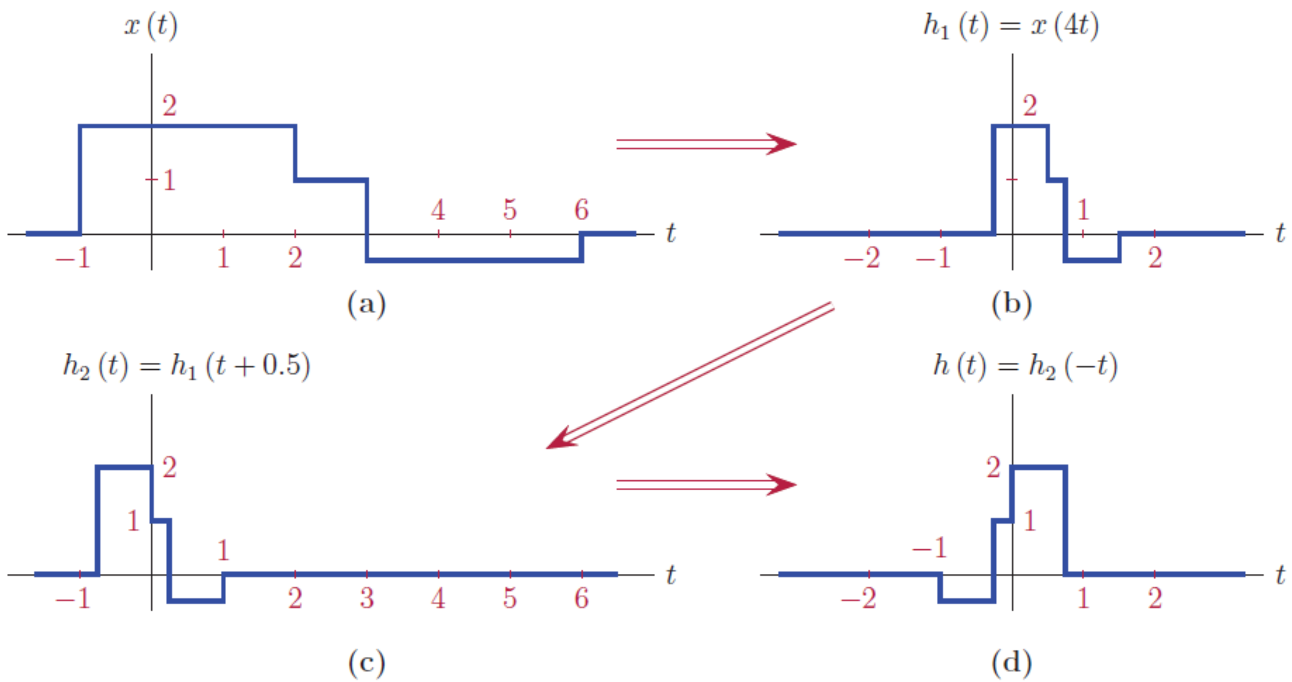


Figure 1.18 – (a) The intermediate signal $h_1(t)$, (b) the intermediate signal $h_2(t)$, and (c) the signal $h(t)$ for Example 1.3.

Signal Operations: Problem 1.4

1.4. For the signal $x(t)$ shown in Fig. P.1.4, compute the following:

a. $g_1(t) = x(-t)$

b. $g_2(t) = x(2t)$

c. $g_3(t) = x\left(\frac{t}{2}\right)$

d. $g_4(t) = x(-t+3)$

e. $g_5(t) = x\left(\frac{t-1}{3}\right)$

f. $g_6(t) = x(4t-3)$

g. $g_7(t) = x\left(1-\frac{t}{3}\right)$

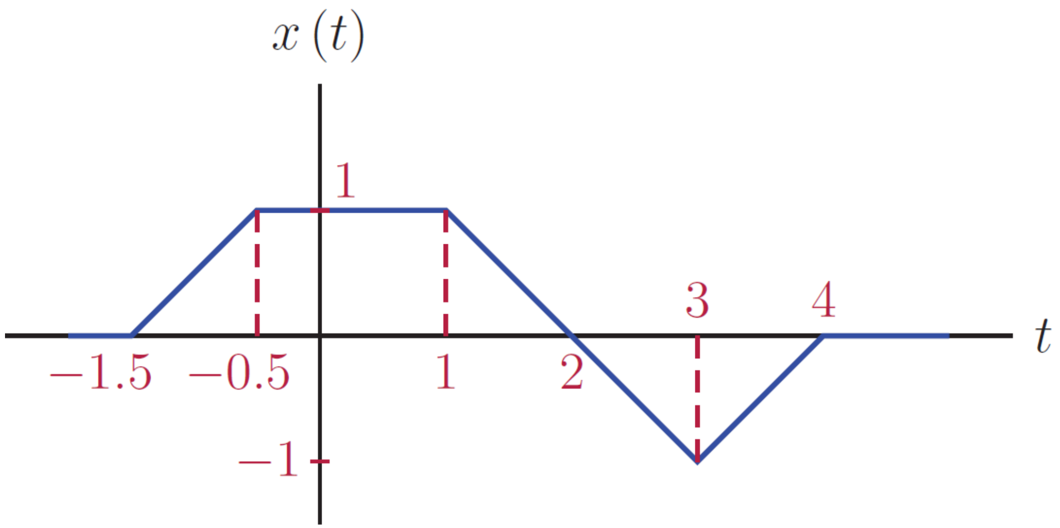
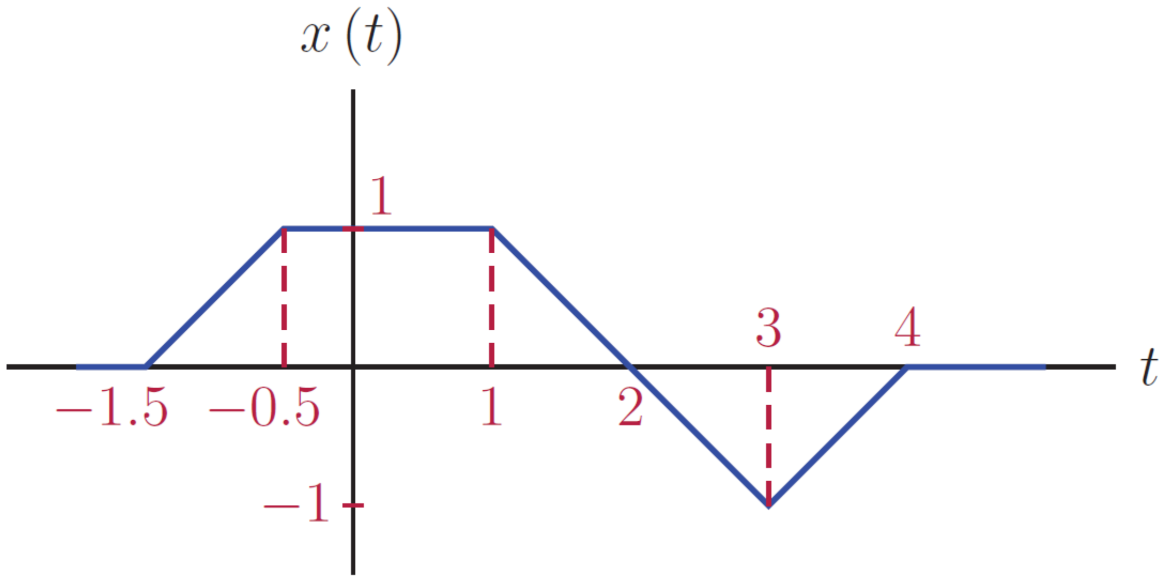


Figure P. 1.4

Signal Operations: Problem 1.4 – Solution

$$x(t)=\begin{cases} t+1.5, & -1.5<t<-0.5 \\ 1, & -0.5<t<1 \\ -t+2, & 1<t<3 \\ t-4, & 3<t<4 \\ 0, & \text{otherwise} \end{cases}$$

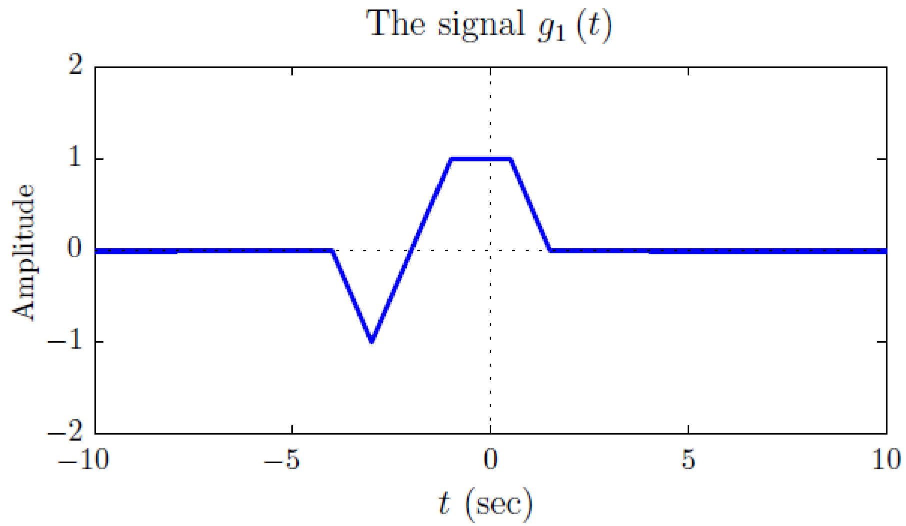


Signal Operations: Problem 1.4 – Solution

a.

Time reversal

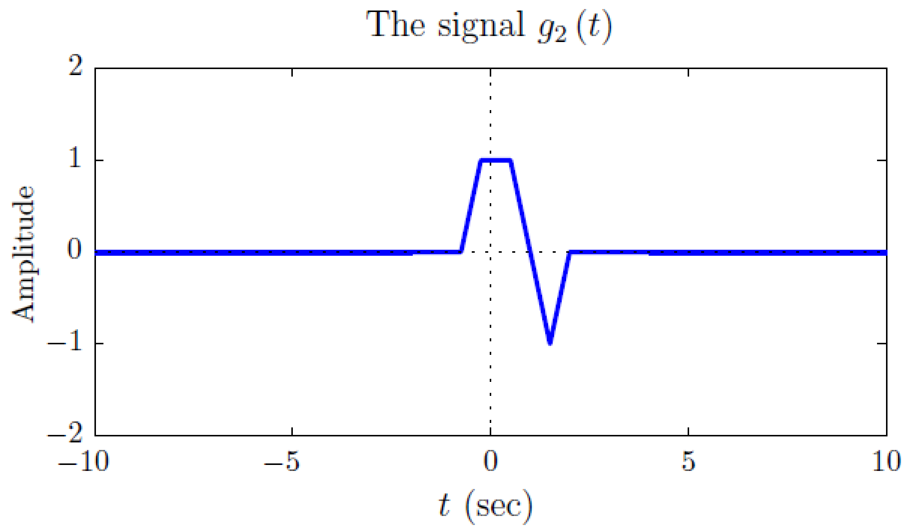
$$g_1(t) = x(-t)$$



b.

Time scaling

$$g_2(t) = x(2t)$$

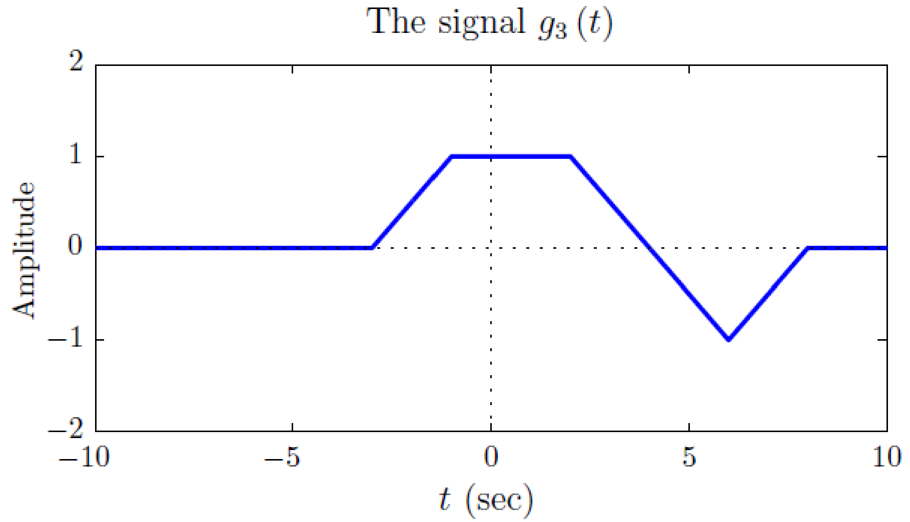


Signal Operations: Problem 1.4 – Solution

c.

Time scaling

$$g_3(t) = x\left(\frac{t}{2}\right)$$



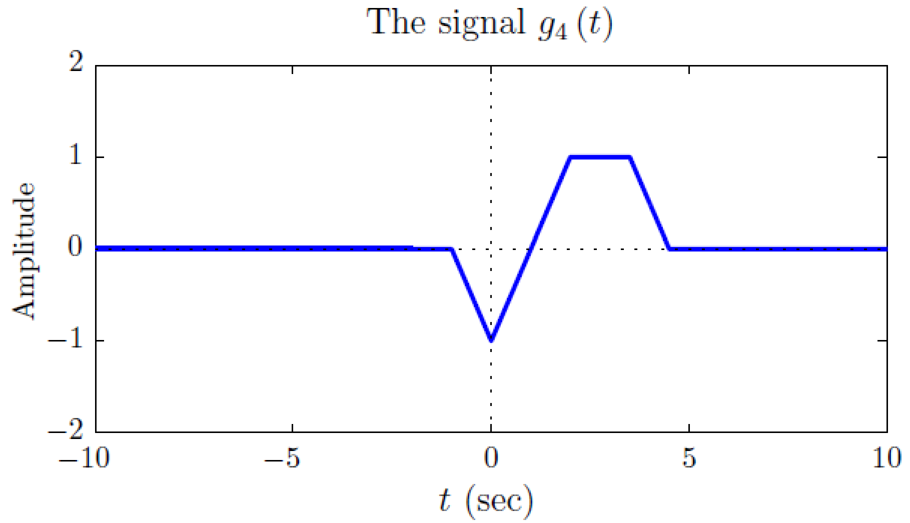
d.

Step 1: Time reversal

$$g_{4a}(t) = x(-t)$$

Step 2: Time shifting

$$g_4(t) = g_{4a}(t-3) = x(-t+3)$$



Signal Operations: Problem 1.4 – Solution

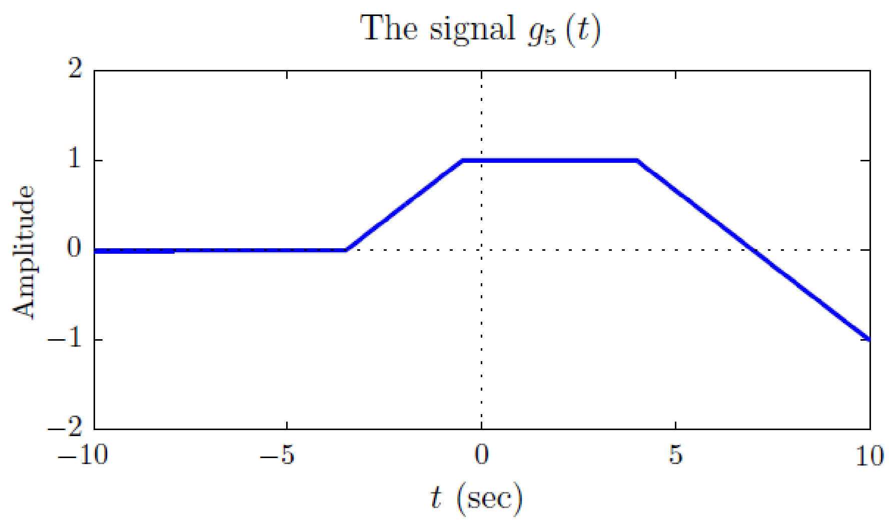
e.

Step 1: Time scaling

$$g_{5a}(t) = x\left(\frac{t}{3}\right)$$

Step 2: Time shifting

$$g_5(t) = g_{5a}(t-1) = x\left(\frac{(t-1)}{3}\right)$$



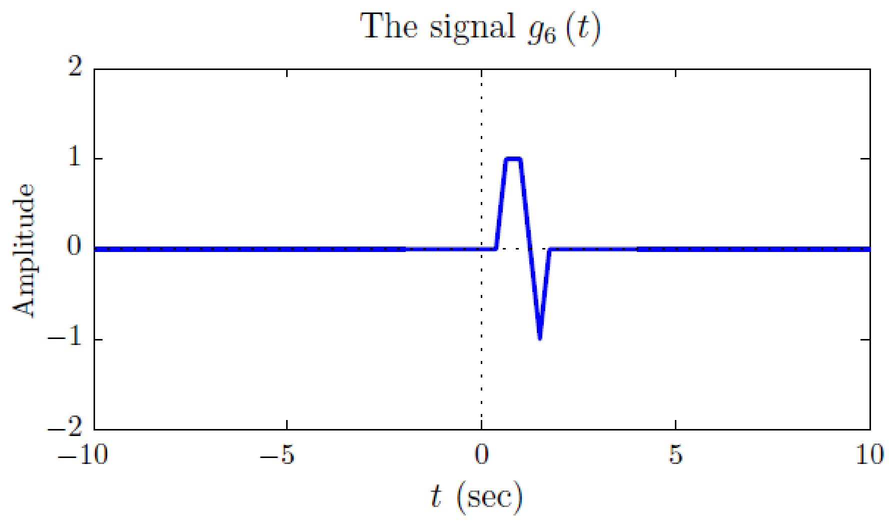
f.

Step 1: Time scaling

$$g_{6a}(t) = x(4t)$$

Step 2: Time shifting

$$g_6(t) = g_{6a}(t-3/4) = x(4t-3)$$



Signal Operations: Problem 1.4 – Solution

g.

$$x(t) = \begin{cases} t+1.5, & -1.5 < t < -0.5 \\ 1, & -0.5 < t < 1 \\ -t+2, & 1 < t < 3 \\ t-4, & 3 < t < 4 \\ 0, & \text{otherwise} \end{cases}$$

$$x(1-\frac{t}{3}) = \begin{cases} 1-\frac{t}{3}+1.5, & -1.5 < 1-\frac{t}{3} < -0.5 \\ 1, & -0.5 < 1-\frac{t}{3} < 1 \\ -\left(1-\frac{t}{3}\right)+2, & 1 < 1-\frac{t}{3} < 3 \\ 1-\frac{t}{3}-4, & 3 < 1-\frac{t}{3} < 4 \\ 0, & \text{otherwise} \end{cases}$$

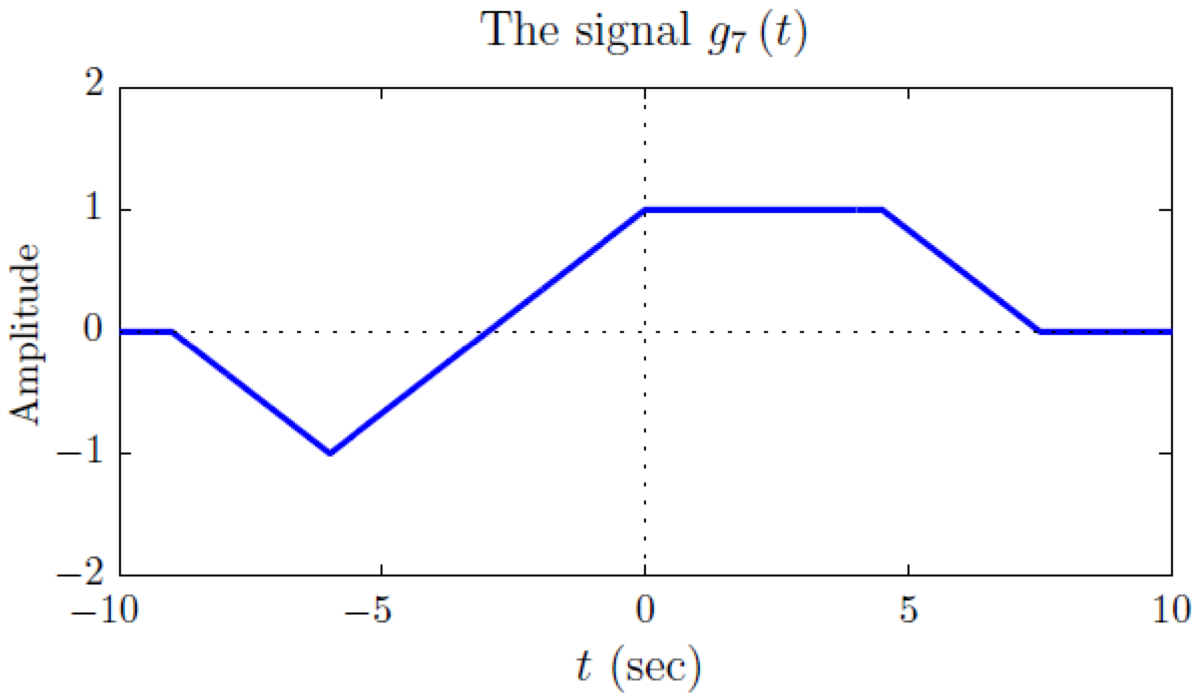
$$x(1-\frac{t}{3}) = \begin{cases} -\frac{t}{3}+2.5, & -2.5 < -\frac{t}{3} < -1.5 \\ 1, & -1.5 < -\frac{t}{3} < 0 \\ \frac{t}{3}+1, & 0 < -\frac{t}{3} < 2 \\ -\frac{t}{3}-3, & 2 < -\frac{t}{3} < 3 \\ 0, & \text{otherwise} \end{cases}$$

$$x(1-\frac{t}{3}) = \begin{cases} -\frac{t}{3}+2.5, & 7.5 > t > 4.5 \\ 1, & 4.5 > t > 0 \\ \frac{t}{3}+1, & 0 > t > -6 \\ -\frac{t}{3}-3, & -6 > t > -9 \\ 0, & \text{otherwise} \end{cases}$$

Signal Operations: Problem 1.4 – Solution

g.

$$x\left(1-\frac{t}{3}\right)=\begin{cases}-\frac{t}{3}+2.5, & 7.5>t>4.5\\ 1, & 4.5>t>0\\ \frac{t}{3}+1, & 0>t>-6\\ -\frac{t}{3}-3, & -6>t>-9\\ 0, & \text{otherwise}\end{cases}$$



Basic Building Blocks For Continuous-Time Signals

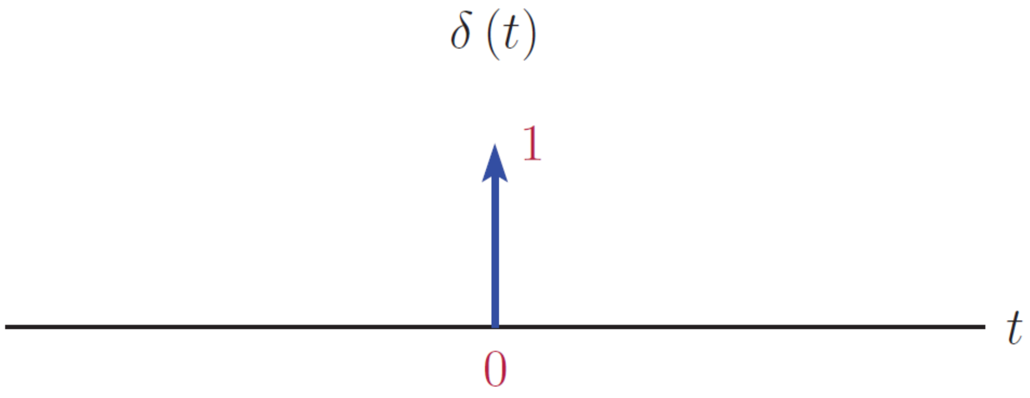
- There are certain basic signal forms that can be used as building blocks for describing signals with higher complexity.
- In this section we will study some of these signals.
- Mathematical models for more advanced signals can be developed by combining these basic building blocks through the use of the signal operations described before.

Unit-Impulse Function

- The unit-impulse function plays an important role in mathematical modeling and analysis of signals and linear systems.

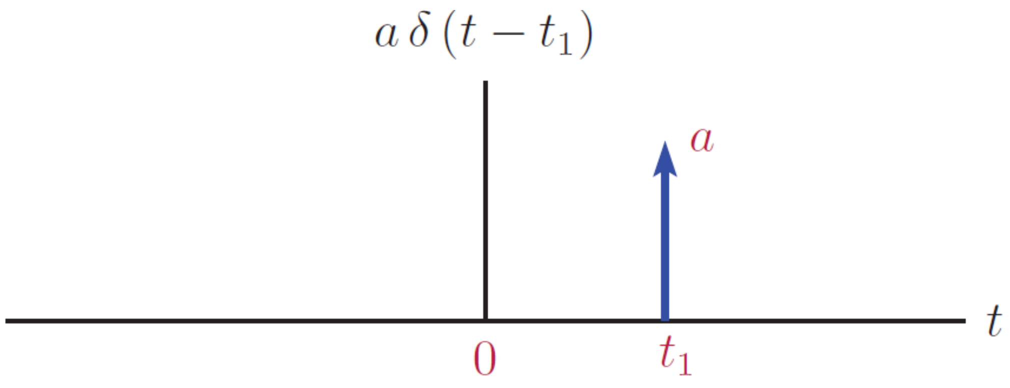
$$\delta(t) = \begin{cases} 0\,, & \text{if } t \neq 0 \\ \text{undefined}\,, & \text{if } t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$



$$a\,\delta(t - t_1) = \begin{cases} 0\,, & \text{if } t \neq t_1 \\ \text{undefined}\,, & \text{if } t = t_1 \end{cases}$$

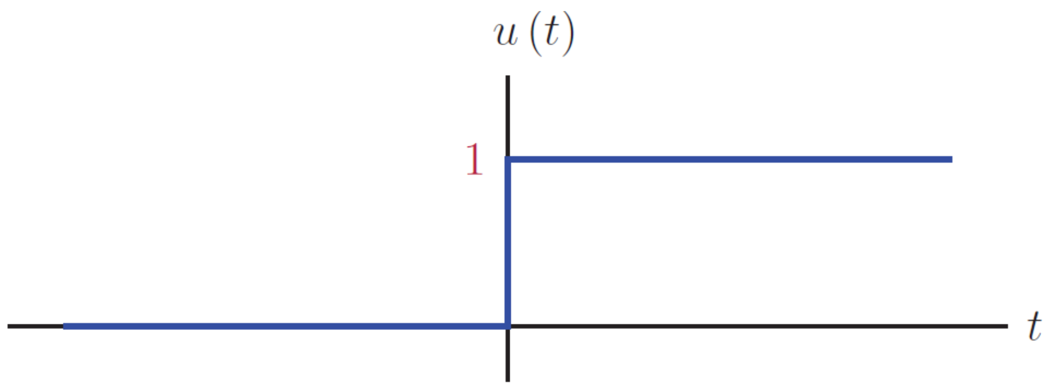
$$\int_{-\infty}^{\infty} a\,\delta(t - t_1)\, dt = a$$



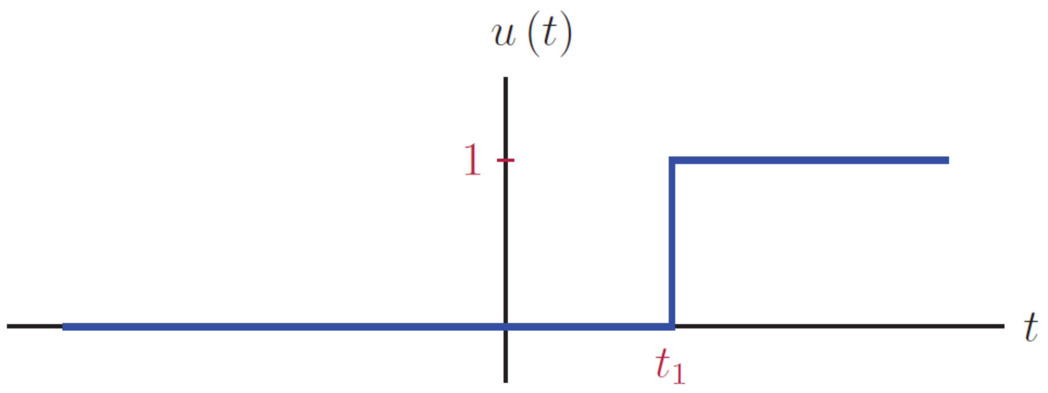
Unit-Step Function

- The unit-step function is useful in situations where we need to model a signal that is turned on or off at a specific time instant.

$$u(t) = \begin{cases} 1\,, & t > 0 \\ 0\,, & t < 0 \end{cases}$$



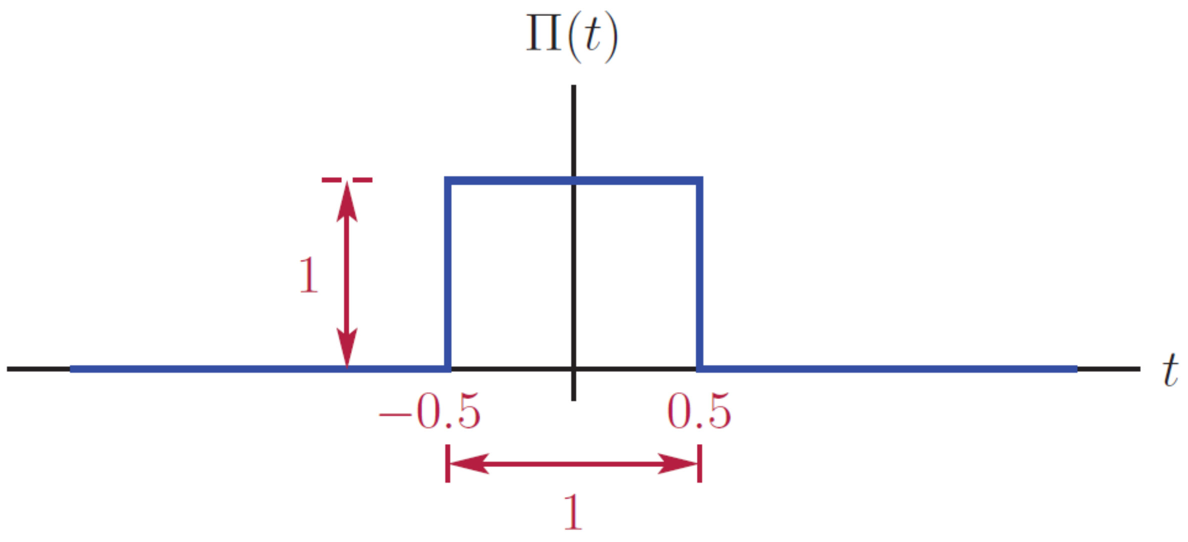
$$u(t - t_1) = \begin{cases} 1\,, & t > t_1 \\ 0\,, & t < t_1 \end{cases}$$



Unit-Pulse Function

- We will define the unit-pulse function as a rectangular pulse with unit width and unit amplitude, centered around the origin.

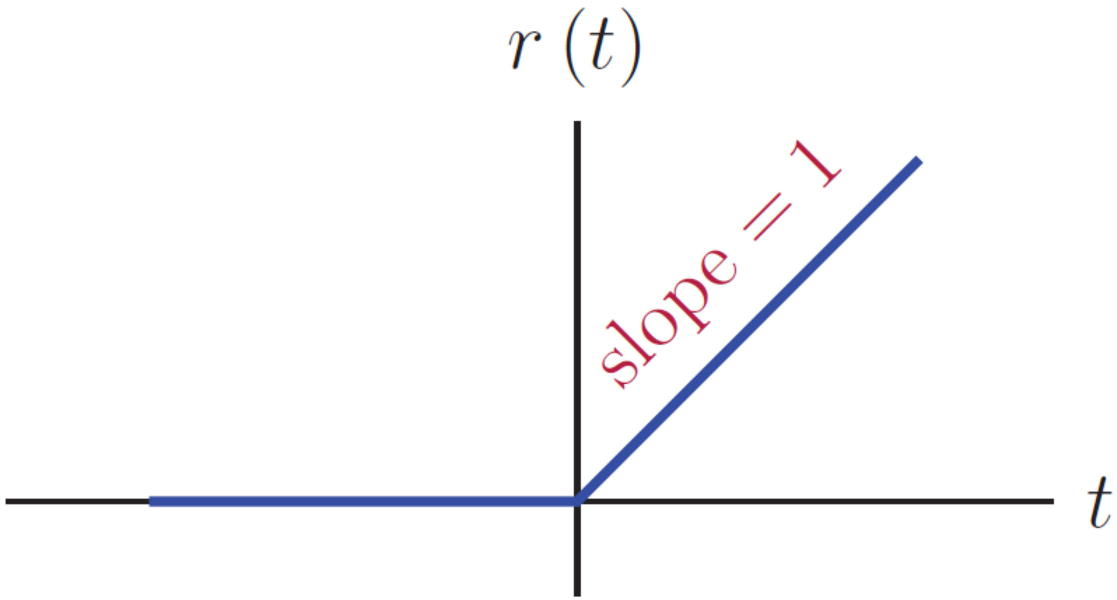
$$\Pi(t) = \begin{cases} 1, & |t| < \frac{1}{2} \\ 0, & |t| > \frac{1}{2} \end{cases}$$



Unit-Ramp Function

- The unit-ramp function has zero amplitude for $t < 0$, and unit slope for $t \geq 0$.

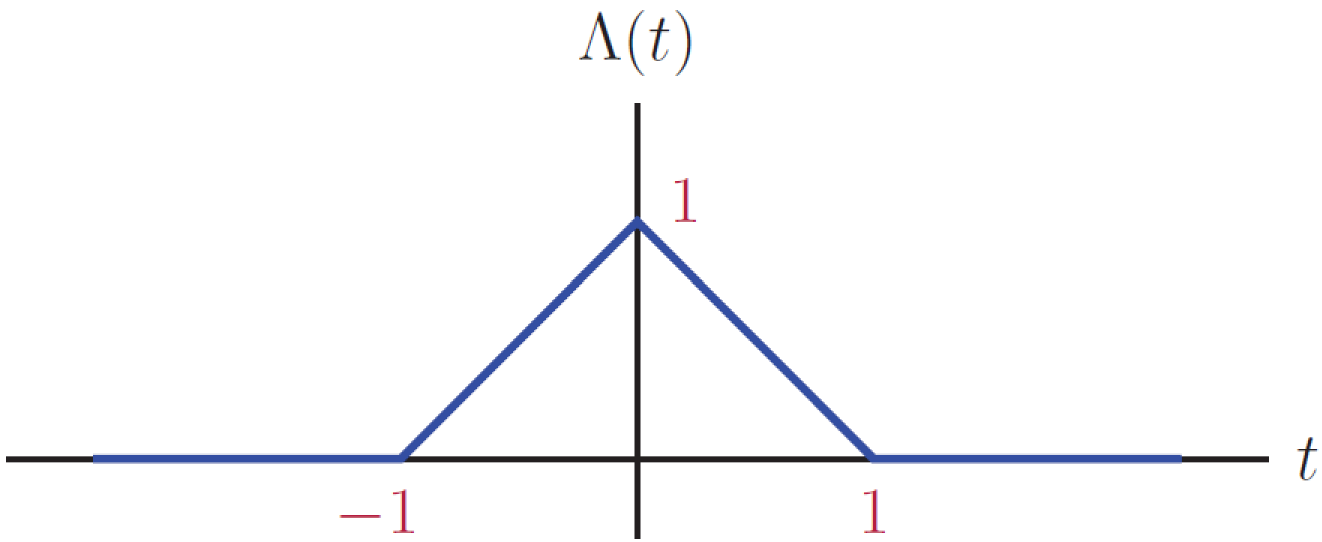
$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



Unit-Triangle Function

- The unit-triangle function is defined as

$$\Lambda(t) = \begin{cases} t+1, & -1 \le t < 0 \\ -t+1, & 0 \le t < 1 \\ 0, & \text{otherwise} \end{cases}$$



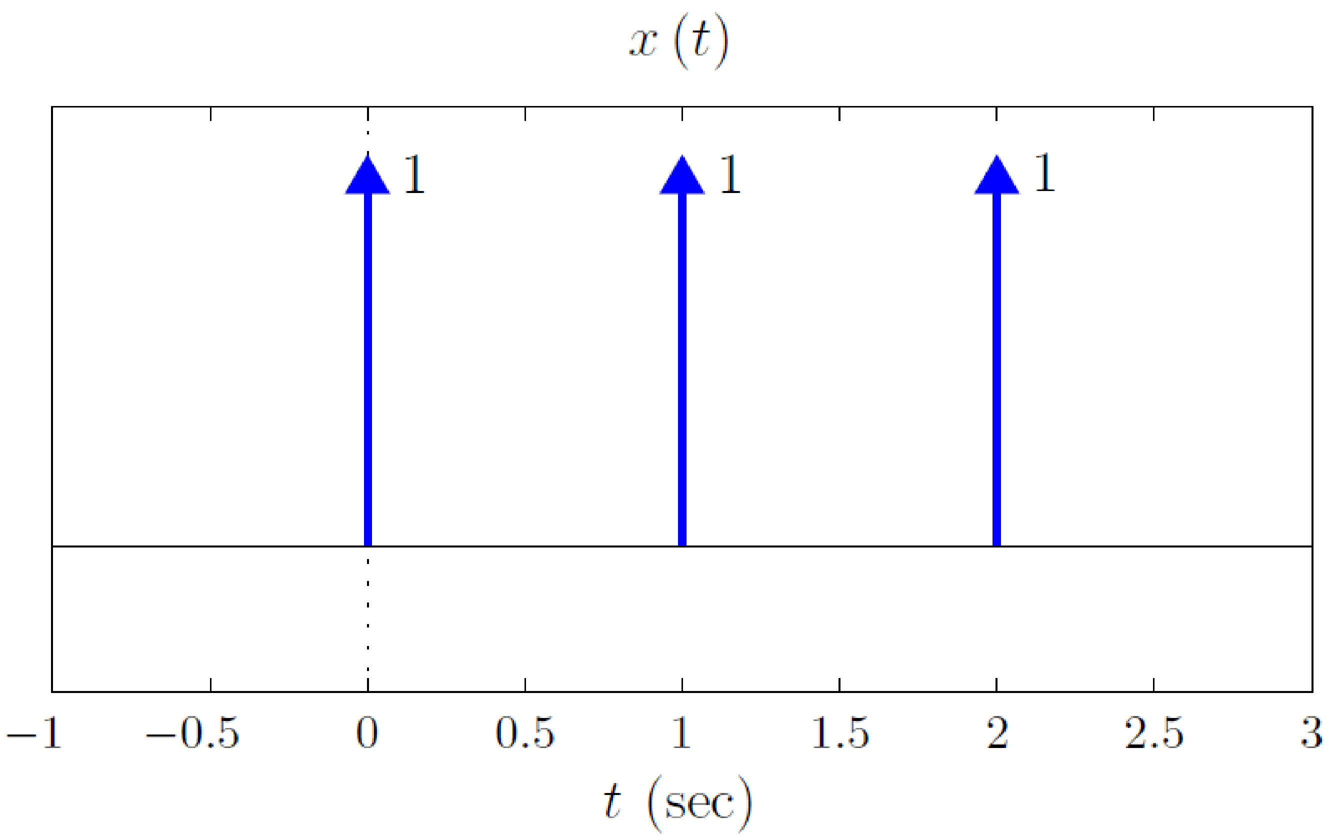
Problem 1.8 (a)

1.8. Sketch each of the following functions.

a. $\delta(t) + \delta(t-1) + \delta(t-2)$

Problem 1.8 (a) – Solution

$$\delta(t) + \delta(t-1) + \delta(t-2)$$



Demos Installation Instructions

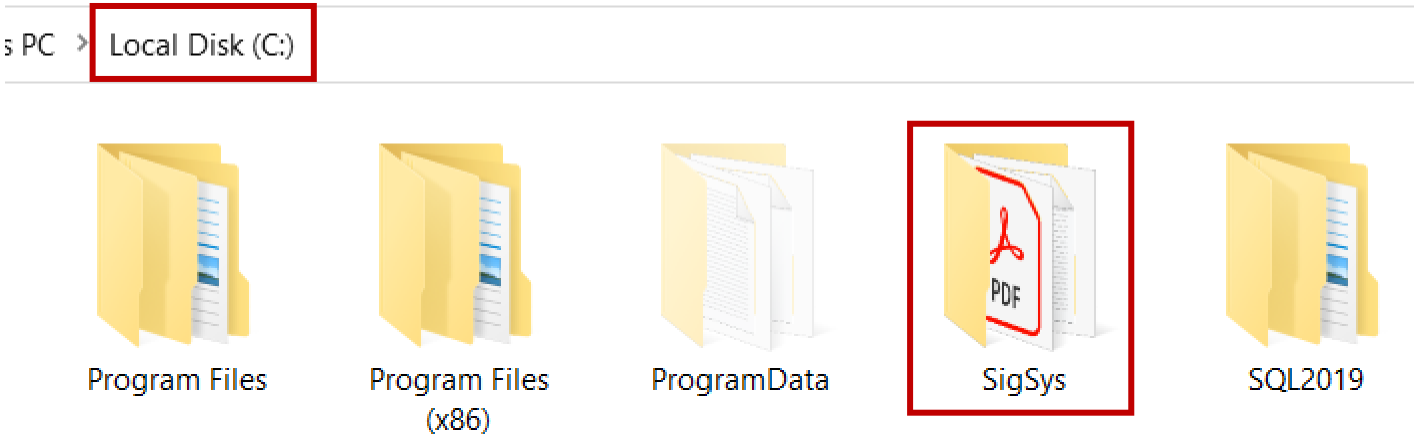
1. Download the current version of the archived file from

   http://www.signalsandsystems.org/downloads

   Download MATLAB files to accompany the book (For MATLAB versions R2014b or newer)

   Detailed installation instructions are here. Alternatively, you can download installation instructions as a pdf file.

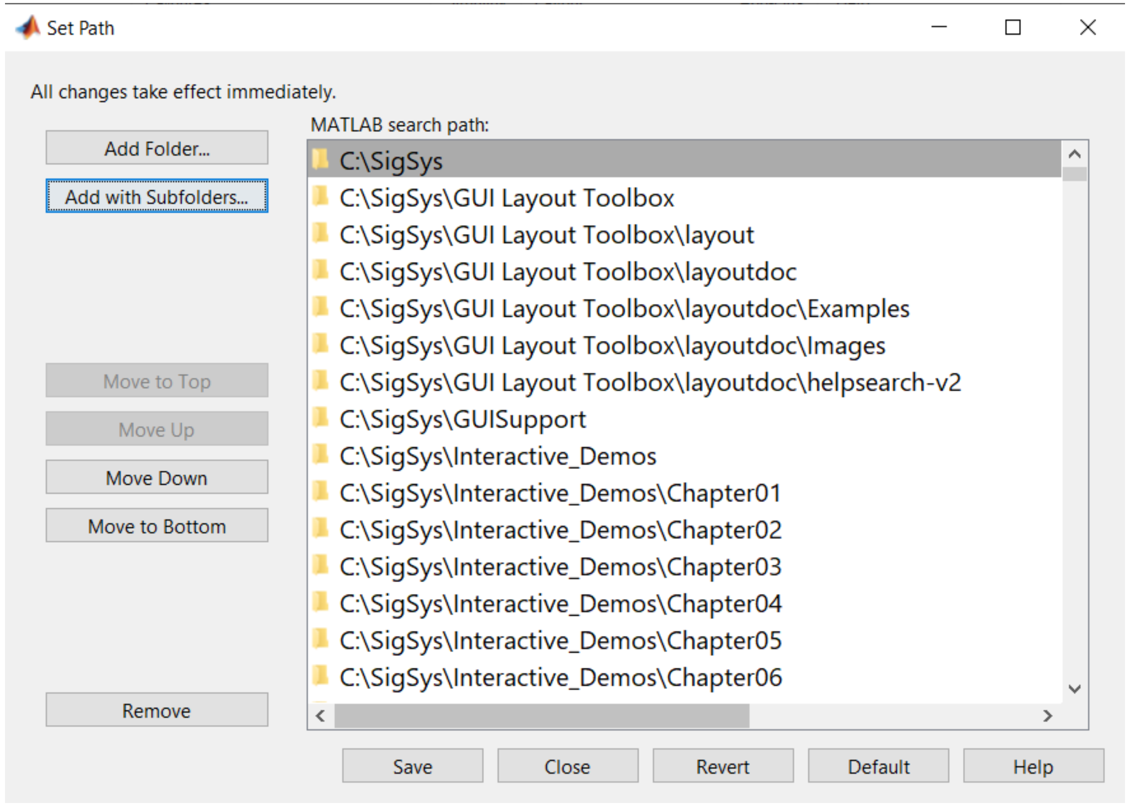
2. Uncompress the archive “`SigSys_MATLAB_v1_03b.zip`”.
3. Copy the folder `SigSys` to a directory such as “`C:\SigSys`”.



Demos Installation Instructions

4. Start MATLAB.
5. In the command window, type the following:

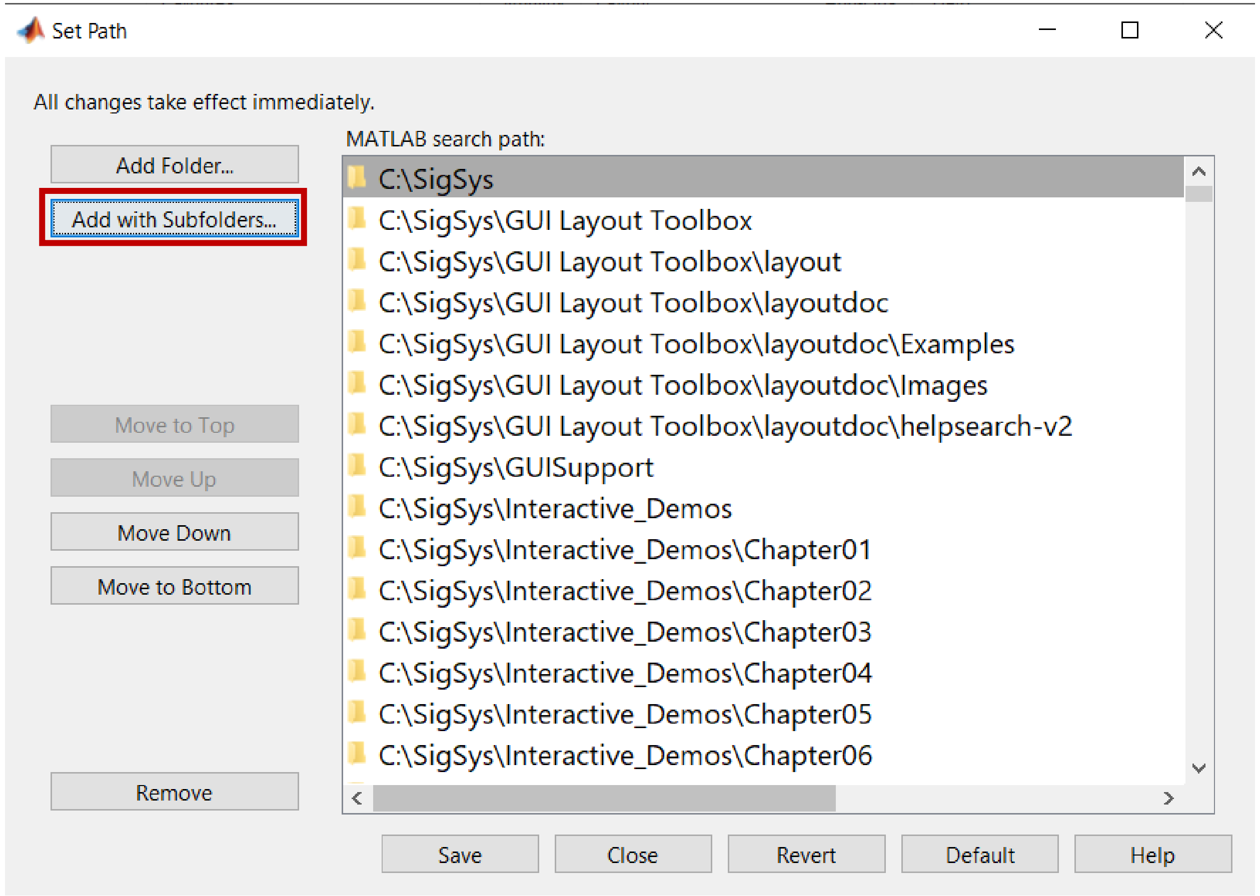
```
>> pathtool
```

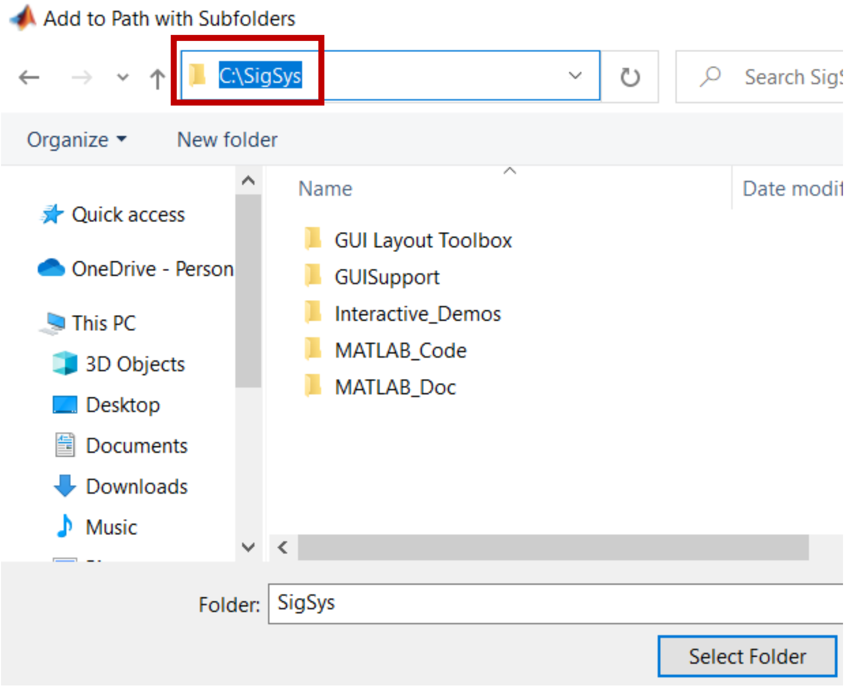


Demos Installation Instructions

6. Click the button “Add with Subfolders…”.

This brings up the “Browse For Folder” dialog shown below:





Demos Installation Instructions

7. Click the button “Select Folder”.
8. Click the “Save” button to close the “Set Path” dialog.